Thermodynamic constraints on equilibrium fluctuations of an order parameter

J. Guioth[1] and D. Lacoste[1]
[1] *Laboratoire de Physico-Chimie Théorique - UMR CNRS Gulliver 7083, PSL Research University, ESPCI, 10 rue Vauquelin, F-75231 Paris, France*

(Dated: May 20, 2016)

We analyze equilibrium fluctuations of a coarse-grained order parameter using variants of isometric fluctuation relations. We focus specifically on the case that fluctuations are coarse-grained due to *local* measurements in a sub-part of a larger system. From these fluctuation theorems, we derive thermodynamic bounds on the fluctuations of *global* or *local* order parameter, which are analogous to uncertainty relations recently derived for current fluctuations. These constraints may be used to infer the value of symmetry breaking field or the relative size of the observed sub-system to the full system using only *local* measurements.

PACS numbers: 05.70.Ln, 05.40.-a 05.70.-a

Away from equilibrium, the second law of thermodynamics quantifies the breaking of the time-reversal symmetry due to energy dissipation, as observed on macroscopic scales. On microscopic scales, the time-reversal symmetry still holds, which manifests itself in the existence of symmetry relations called fluctuation relations. These relations constrain the probability distributions of thermodynamic quantities arbitrarily far from equilibrium. Their discovery [1–6] represents a major progress in our understanding of the second law of thermodynamics, which has furthermore triggered intense experimental activity on small non-equilibrium systems.

All these developments have put a strong emphasis on dissipative systems, despite the fact that the formalism is more general and applicable to other forms of symmetry breaking even at equilibrium. For instance, a system of N Ising spins in a magnetic field is a classic illustration of an equilibrium system with discrete symmetry breaking. In discussing this pedagogical example [7], N. Goldenfeld derived a simple relation for the ratio of the probability to observe a magnetization $\mathbf{M}_N$, $P_{\mathbf{B}}(\mathbf{M}_N)$ with the probability to observe instead $-\mathbf{M}_N$:

$$P_{\mathbf{B}}(\mathbf{M}_N) = P_{\mathbf{B}}(-\mathbf{M}_N)\ \mathrm{e}^{2\beta \mathbf{B}\cdot\mathbf{M}_N}. \tag{1}$$

The similarity of Eq. (1) with the Gallavotti-Cohen fluctuation theorem has only been noticed recently [4, 8]. Inspired by these works and by the discovery of fluctuation relations combining spatial and time-reversal symmetries called isometric fluctuation relations [9], one of us derived an extension of Eq. (1) for general symmetries described by group theory, which was then illustrated on a number of classic models of statistical physics [10, 11].

In this work, another family of relations, closer to the Crooks Fluctuation theorem [3], has been also derived. This second type of relation compares the probability distribution of the order parameter in the absence and in the presence of the magnetic field:

$$\frac{P_B(\mathbf{M}_N)}{P_0(\mathbf{M}_N)} = e^{\beta(\mathbf{B}\cdot\mathbf{M}_N - \Delta F)}. \tag{2}$$

where $\Delta F = F_N(0) - F_N(\mathbf{B})$ is the difference of free energy between a state in which $\mathbf{B} = 0$ and the state with a magnetic field $\mathbf{B}$.

The use of Eqs. (1) and (2) requires statistics of the fluctuations of the *global* order parameter of the ensemble of the N spins. If only partial information is available, which occurs when only part of the system is probed *locally*, the relations Eq. (1) and Eq. (2) fail. A similar situation arises out of equilibrium due to coarse-graining [12–18]. Understanding how to extract relevant information in such cases is quite pertinent experimentally even at equilibrium since *local* measurements are often the only choice, in the frequent case that the system is just too big to be analyzed globally. The purpose of the present letter is to show that a proper analysis of fluctuations allows to estimate the relative size ratio of the observation window to the full system or the free energy of the large system.

In order to study a *local* version of the relations of Eq. (1) and Eq. (2), we consider a subset of the N spins containing $n < N$ spins only, $\Lambda = \{\boldsymbol{\sigma}_i\}_{i=1}^n$, with magnetization $\mathbf{M}_n(\boldsymbol{\sigma}) = \sum_{i=1}^n \boldsymbol{\sigma}_i$. The remaining spins $\bar{\Lambda} = \{\boldsymbol{\sigma}_i\}_{i=n}^N$ play the role of an "environment" for the spins of Λ. This environment has a magnetization $\overline{\mathbf{M}}_n(\boldsymbol{\sigma}) = \sum_{i=n}^N \boldsymbol{\sigma}_i$, so that $\mathbf{M}_N = \mathbf{M}_n + \overline{\mathbf{M}}_n$. The local equivalent of Eq. (1) is:

$$P_{\mathbf{B}}(\mathbf{M}_n) = P_{\mathbf{B}}(-\mathbf{M}_n)\ \mathrm{e}^{\beta(2\mathbf{B}\cdot\mathbf{M}_n + \Gamma_{\mathbf{B}}(\mathbf{M}_n))}. \tag{3}$$

We have introduced the function

$$\begin{aligned}\Gamma_{\mathbf{B}}(\mathbf{M}_n) &= k_B T \ln\langle e^{-2\beta\mathbf{B}\cdot\overline{\mathbf{M}}_n} | -\mathbf{M}_n\rangle_{\mathbf{B}}, \qquad (4)\\ &= k_B T \ln \int e^{-2\beta\mathbf{B}\cdot\overline{\mathbf{M}}_n}\,\mathrm{Proba}(\overline{\mathbf{M}}_n| -\mathbf{M}_n) d^3\overline{\mathbf{M}}_n,\end{aligned}$$

where $\mathrm{Proba}(\overline{\mathbf{M}}_n| -\mathbf{M}_n)$ denotes the conditional probability of $\overline{\mathbf{M}}_n$ given a magnetization $-\mathbf{M}_n$ for the subpart. Thus, $\Gamma_{\mathbf{B}}(\mathbf{M}_n)$ is a correction factor which quantifies the failure of Eq. (1) due to the reduction of available information in the fluctuations. By construction, this factor must be an odd function of $\mathbf{M}_n$, *i. e.*: $\Gamma_{\mathbf{B}}(-\mathbf{M}_n) = -\Gamma_{\mathbf{B}}(\mathbf{M}_n)$.

Similarly, for Eq. (2), one obtains

$$\frac{P_B(\mathbf{M}_n)}{P_0(\mathbf{M}_n)} = e^{\beta(\mathbf{B}\cdot\mathbf{M}_n - \Delta F + \Omega_{\mathbf{B}}(\mathbf{M}_n))}, \tag{5}$$

where $\Omega_{\mathbf{B}}$ is the correction factor,

$$\Omega_{\mathbf{B}}(\mathbf{M}_n) = k_B T \ln\langle e^{-\beta \mathbf{B}\cdot\overline{\mathbf{M}}_n} | -\mathbf{M}_n\rangle_0, \tag{6}$$

which unlike $\Gamma_{\mathbf{B}}$ does not need to be an odd function of $\mathbf{M}_n$. Note that the two correction factors are expressed in terms of two different averages, $\langle..\rangle_{\mathbf{B}}$ denotes the average with respect to the full Hamiltonian including the magnetic field while $\langle..\rangle_{\mathbf{0}}$ refers to an average with respect to the Hamiltonian at $\mathbf{B} = \mathbf{0}$.

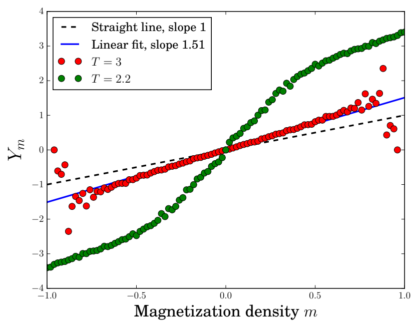

FIG. 1. Asymmetry function Y_m versus magnetization density m for the 2D Ising model in a magnetic field, at a temperature $T = 3$ (above T_c) or $T = 2.2$ (below T_c). While this function is always a straight line of slope one according to Eq. (1) when evaluated for the full system (dashed line), it differs from it when evaluated for a sub-part (symbols). Here the sub-part contains $n = 100$ spins, the full system $N = 400$ spins, and the magnetic field is $B = 0.01$. The critical temperature is $T_c \simeq 2.38$ for this system size.

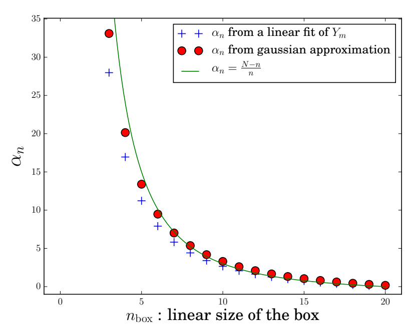

FIG. 2. Parameter α_n versus the size of the box n_{box} (with $n = n_{box}^2$) for $T \simeq T_c^+$ and a small magnetic field $B = 0.005$. Two different estimates for α_n are shown, either from a linear fit of Y_m (blue plus symbols) or more directly using a Gaussian approximation of the fluctuations (red circles), while the green solid line corresponds to $(N-n)/n$.

In the case that all the spins of the sub-part and the rest are independent, there are no correlations between Λ and $\bar{\Lambda}$, which means Proba$(\overline{\mathbf{M}}_n|-\mathbf{M}_n) = $ Proba$(\overline{\mathbf{M}}_n)$. Using Jarzynski like relations deduced from Eqs. (1)-(2) for the complementary part, one has immediately $\Gamma_{\mathbf{B}}(\mathbf{M}_n) = \Omega_{\mathbf{B}}(\mathbf{M}_n) = 0$. Therefore, the breaking of the Fluctuation relations Eqs. (1)-(2) arises entirely from the correlations between the domains Λ and $\bar{\Lambda}$. In this paper, we focus mainly on applications of Eq. (3), while some applications of Eq. (5) are discussed briefly in [19].

In order to simplify the evaluation of these correlations, we can formally split the domain $\bar{\Lambda}$ into a subset of strongly correlated spins in the immediate vicinity of Λ which we assume contribute linearly to the magnetization of $\bar{\Lambda}$, and the rest of the spins, which are less correlated and may be treated as the ideal part of the environment from the point of view of the system [14]. In other words:

$$\overline{\mathbf{M}}_n = \alpha_n \mathbf{M}_n + \xi_n. \tag{7}$$

We allow α_n to depend on the magnetization $\mathbf{M}_n$, but we require that ξ_n be uncorrelated with $\mathbf{M}_n$. Due to this latter assumption, it is easy to show that α_n is the following normalized co-variance between $\mathbf{M}_n$ and $\overline{\mathbf{M}}_n$:

$$\alpha_n = \frac{\langle \mathbf{M}_n \cdot \overline{\mathbf{M}}_n\rangle - \langle \mathbf{M}_n\rangle \cdot \langle \overline{\mathbf{M}}_n\rangle}{\langle \mathbf{M}_n^2\rangle - \langle \mathbf{M}_n\rangle^2}. \tag{8}$$

Then, using Eq. (4) and Eq. (7), one finds a linear correction $\Gamma_{\mathbf{B}}(\mathbf{M}_n) = 2\alpha_n \mathbf{M}_n \cdot \mathbf{B}$. Let us introduce the magnetization density $m = \mathbf{M}_n \cdot \hat{z}/n$, with $\mathbf{B} = B\hat{z}$ and the asymmetry function Y_m defined by

$$Y_m = \frac{1}{2\beta B n} \ln \frac{P_{\mathbf{B}}(m)}{P_{\mathbf{B}}(-m)}. \tag{9}$$

In this simple case, the asymmetry function for the *local* order parameter is a straight line of slope $1 + \alpha_n$, instead of slope one for the *global* order parameter. When α_n does not depend on the magnetization, the change of slope could be described by the inverse effective temperature $\beta_{eff} = \beta(1 + \alpha_n)$ or by an effective magnetic field, similarly to the nonequilibrium case [12]. Since the magnetization of $\bar{\Lambda}$ acts like a field for Λ enhancing its magnetization, $\alpha_n \geq 0$ and this effective temperature is smaller than T.

A straight asymmetry function with a slope larger than one is indeed found in figure 1, for the fluctuations of the order parameter in a box of $n = 100$ spins among a total of $N = 400$ spins at the temperature $T = 3$. At this temperature, well above T_c, the correlation length should be small so that the spins of $\bar{\Lambda}$ should indeed be uncorrelated with that of Λ except for those at the interface between both domains, thus justifying the assumptions made in introducing Eq. (7).

As $T \to T_c^+$, the correlation length increases until it becomes of the order of the size of the full system, at which point it is necessarily larger than the sub-part. As the critical point is approached, one needs to consider a correspondingly reduced magnetic field, in order to keep the energy of the symmetry breaking field finite and of the order of k_BT, so as to allow statistically reversals of the magnetization. In such a correlated system, the contribution of ξ_n in Eq. (7) should vanish on average, and the average magnetization density is $\langle \mathbf{m} \rangle = \langle \mathbf{M}_n \rangle / n = \langle \overline{\mathbf{M}}_n \rangle / (N-n)$, which implies $\alpha_n \simeq (N-n)/n$. Note that away from the critical point, α_n also scales as $1/n$ but the prefactor does not have such a simple form. We have checked that this scaling of α_n as function of n is indeed approached when getting closer to the critical point as shown in fig 2. This parameter α_n may be determined from a linear fit of the asymmetry function Y_m as explained above or more directly as a ratio of the variance to the mean. Alternatively, one could use Eq. (8) to determine it. All these determinations are consistent with each other in the regime $T \geq T_c$ and when the statistics is not limiting. Using any of these methods to determine α_n near the critical point, one can infer from this parameter the relative size of the observation window to the size of the large system using the relation $\alpha_n \simeq (N-n)/n$.

Below the critical point, the asymmetry function of the *local* order parameter, has a sigmoidal shape as shown in the curve corresponding to the temperature $T = 2.2$ in fig. 1.

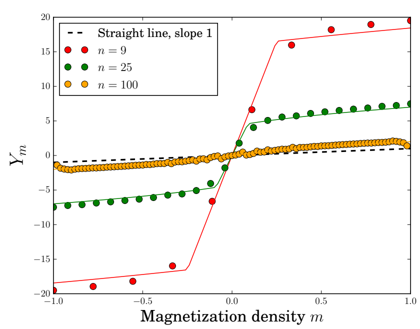

FIG. 3. Asymmetry function for the Curie-Weiss model versus magnetization density m for various box sizes $n = 3 \times 3$, 5×5 or 10×10 for a full system containing $N = 15 \times 15$ spins in a magnetic field $B = 0.01$ and at the temperature $T = 0.8$. The symbols are the Monte Carlo simulation data and the solid line is the semi-analytic evaluation using Eq. (12).

In order to better understand the shape of this function, we now consider the mean field Curie-Weiss model with N Ising spins [8]. The distribution of the *local* order parameter is

$$P_{\mathbf{B}}(\mathbf{M}_n) = \frac{1}{Z_N(\mathbf{B})} \sum_{\boldsymbol{\sigma}} \mathrm{e}^{\frac{\beta J}{2N} \mathbf{M}_N^2 + \beta \mathbf{B} \cdot \mathbf{M}_N} \delta \left[\mathbf{M}_n - \mathbf{M}_n(\boldsymbol{\sigma}) \right], \tag{10}$$

where the sum is taken over configurations of the full system. After separating the contribution of spins belonging to Λ and to $\bar{\Lambda}$, one obtains

$$P_{\mathbf{B}}(\mathbf{M}_n) = \frac{Z_{\bar{\Lambda}}(\mathbf{B} + J\mathbf{M}_n/N)}{Z_N(\mathbf{B})} \mathrm{e}^{\frac{\beta J}{2N} \mathbf{M}_n^2 + \beta \mathbf{B} \cdot \mathbf{M}_n} C_n(\mathbf{M}_n), \tag{11}$$

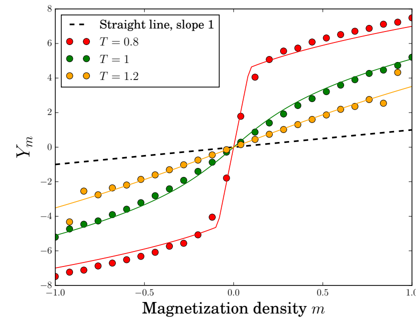

FIG. 4. Asymmetry function for the Curie-Weiss model versus the magnetization density m for temperatures $T = 0, 8$, $T = 1$ and $T = 1, 2$ for a box $n = 5 \times 5$. The other parameters are as in Fig. 3.

where $Z_{\bar{\Lambda}}(\mathbf{B} + J\mathbf{M}_n/N)$ is the partition function of an ensemble of $N - n$ spins of $\bar{\Lambda}$, subject to the effective magnetic field $\mathbf{B}_{eff} = \mathbf{B} - J\mathbf{M}_n/N$ and interacting with the effective coupling constant $J_{eff} = J(N-n)/N$. The function $C_n(\mathbf{M}_n)$ is the number of microstates with a given value of local magnetization $\mathbf{M}_n$ [10]. Since this function only depends on $|\mathbf{M}_n|$, one obtains for the correction factor of Eq. (3):

$$\Gamma_{\mathbf{B}}(\mathbf{M}_n) = F_{\bar{\Lambda}}(\mathbf{B} - J\mathbf{M}_n/N) - F_{\bar{\Lambda}}(\mathbf{B} + J\mathbf{M}_n/N), \tag{12}$$

where $F_{\bar{\Lambda}}$ is the mean-field free energy associated with the partition function $Z_{\bar{\Lambda}}$.

In figure 3, Monte Carlo simulation results for the asymmetry function Y_m are shown versus m for various box sizes, where m now denotes the magnetization density of the subpart rather than that of the full system. As expected, the asymmetry function tends towards a straight line of slope one in the limit that the box becomes the full system. More importantly, for small box sizes, these simulation results are in good agreement with the semi-analytical prediction based on the direct evaluation of Eq. (12). This evaluation becomes exact in the limit of large $N - n$.

Interestingly, the asymmetry function present two cusps, symmetrically located on either side of the origin, which are consequences of the singularities present in the derivative of the free energy $F(\mathbf{B})$ when the thermodynamic limit is taken before the limit $\mathbf{B} \to 0$ [10]. Indeed, the locations of these cups are obtained from the condition that the two free energies $F_{\bar{\Lambda}}(\mathbf{B} \pm J\mathbf{M}_n/N)$ in Eq. (12) have a vanishing argument, which happens when $m = m^* = \pm NB/Jn$. These cusps can be seen in figure 3, they move away from the origin as the box size decreases. Since $N - n$ is not very large in the simulation, the simulation data is smoother near the cusps than the analytical expression due to finite size effects.

In the thermodynamic limit for the full system, namely when $n/N \to 0$ at finite non-zero B, the asymmetry function should become again a straight line of slope one, as can be seen directly shown by taking such a limit directly in Eq. (12). The approach of this limit is not visible in figure 3, because when the box size is very small, the simulation data points become very sparse due to the reduction of the accessible values of the coarse-grained magnetization.

The curves at fixed box size (smaller than the full system) for different temperatures confirm the results obtained previously for the Ising model in fig. 1, namely that the asymmetry function is a straight line above T_c but is a sigmoidal function below T_c. To summarize, we have analyzed various consequences of the *local* fluctuation relation of Eq. (3), which represents a first main result.

We now go back to the general case (not necessarily mean-field) and investigate further consequences. These will take the form of bounds for the fluctuations, analogous to relations recently derived in the non-equilibrium case [20, 21]. Let us start with the case of *global* fluctuations for the full system. We recall the definition of the large deviation function:

$$P_{\mathbf{B}}(\mathbf{M}_N) \simeq e^{-N\Phi_{\mathbf{B}}(\mathbf{m})}, \tag{13}$$

for N large and $\mathbf{m} = \mathbf{M}_N/N$. Let us introduce the following quadratic approximation of $\Phi_{\mathbf{B}}(\mathbf{m})$

$$\Phi_{LR}(\mathbf{m}) = \beta \frac{(\mathbf{m}-\mathbf{m}^*)^2 \mathbf{B}\cdot\mathbf{m}^*}{2(\mathbf{m}^*)^2}, \tag{14}$$

where $\mathbf{m}^*$ is the most probable value of the magnetization which is such that $\Phi_{\mathbf{B}}(\mathbf{m}^*) = \Phi'_{\mathbf{B}}(\mathbf{m}^*) = 0$. Using the fluctuation theorem of Eq. (1), it is easy to verify that $\Phi(\mathbf{m})$ and $\Phi_{LR}(\mathbf{m})$ take the same value and their derivatives are equal at the two symmetrically placed points $\pm\mathbf{m}^*$.

Along these lines, we propose the following bound for $\Phi_{\mathbf{B}}(\mathbf{m})$,

$$\Phi_{\mathbf{B}}(\mathbf{m}) \geq \Phi_{LR}(\mathbf{m}), \tag{15}$$

which is illustrated for the mean-field Curie-Weiss model in figure 5 (see also Suppl. for details [19]). We now denote m the projection of $\mathbf{m}$ along $\mathbf{B}$. Note that since $\Phi_{\mathbf{B}}(\mathbf{m})$ is convex, the part of the curve between $[-m^*, m^*]$ with negative curvature should be replaced by a straight line following the common tangent construction [22].

To prove this bound, we first note that Eq. (15) is implied by

$$\Phi''_B(m^*) \geq \frac{\beta B}{m^*}, \tag{16}$$

since $\Phi''_{\mathbf{B}}(\mathbf{m})$ is convex. Furthermore, it follows from $\Phi_{\mathbf{B}}(\mathbf{m}) = \Phi_0(\mathbf{m}) - \beta B m$ [10, 11], that $\Phi'_{\mathbf{B}}(\mathbf{m}^*) = \Phi'_0(\mathbf{m}^*) - \beta B = 0$. Therefore, the inequality Eq. (16) is equivalent to the function $h(m) = \Phi''_0(m) - \Phi'_0(m)/m$ been positive. This is indeed the case because $\Phi'_0(m)$ is strictly convex for $m > 0$ and strictly concave for $m < 0$ [19]. Further, both Eqs. (15)-(16) become saturated as $B \to 0$, since the equality which is obtained in that case is nothing but the well-known linear response relation [7]. This justifies the name of the index LR in Φ_{LR}, which stands for the linear response regime with respect to $\mathbf{B}$ [21].

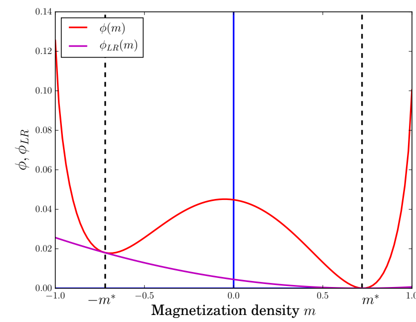

FIG. 5. Large deviation function $\Phi_{\mathbf{B}}(\mathbf{m})$ and its linear-response approximation $\Phi_{LR}(\mathbf{m})$ for the *global* fluctuations of the magnetization density $\mathbf{m}$ in the Curie-Weiss model for a temperature $T = 0.8$ (in the ferromagnetic phase) and a magnetic field $B = 0.01$. The two functions are tangent at the points $\pm m^*$ located by the two vertical dashed lines.

Let us now look at some consequences of Eq. (15). The variance of the order parameter is $\Phi''_{\mathbf{B}}(\mathbf{m}) = 1/(N\mathrm{Var}(\mathbf{m}))$. Then Eq. (15) translates to the second derivative because $\Phi_{\mathbf{B}}$ and Φ_{LR} have the same minimum. Thus,

$$\frac{\mathrm{Var}(\mathbf{m})}{\langle m\rangle} \leq \frac{k_B T}{BN}, \tag{17}$$

where $\mathbf{m}^* = \langle \mathbf{m}\rangle$. Note that this inequality implies $\mathrm{Var}(M_N)/\langle M_N\rangle \leq k_B T/B$ for the extensive magnetization. It is important to appreciate that this bound holds generally is not limited to the linear response regime of small magnetic field. It also holds at any temperature and any Hamiltonian for which the isometric fluctuation relation holds.

Let us look at similar properties but now for *local* fluctuations of the order parameter in a sub-part instead of *global* fluctuations. Now the large deviation function is defined as

$$P_{\mathbf{B}}(\mathbf{M}_n) \simeq e^{-n\phi_{\mathbf{B}}(\mathbf{m})}, \tag{18}$$

for n sufficiently large and now the magnetization density is $\mathbf{m} = \mathbf{M}_n/n$.

In view of the modified fluctuation theorem of Eq. (3), the approximation

$$\phi_{LR}(\mathbf{m}) = \beta B Y_{m^*} \frac{(\mathbf{m}-\mathbf{m}^*)^2}{2(\mathbf{m}^*)^2}, \tag{19}$$

is correct by construction close to $\mathbf{m} = \mathbf{m}^*$ and has the expected value at $\mathbf{m} = -\mathbf{m}^*$ but unlike Φ_{LR} may not have the correct tangent at this point. Fortunately, when

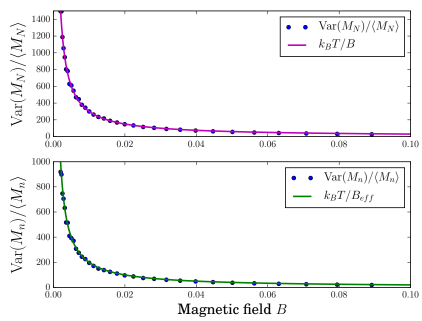

FIG. 6. Numerical test of the bounds for *global* fluctuations given by Eq. (17) and for *local* fluctuations given by Eq. (20) for the 2D-Ising model at a temperature $T = 3.0$ (in the paramagnetic phase) as a function of the magnetic field B. The size ratio for the sub-part to full system and the value of α_n are as in fig. 1.

the asymmetry function Y_m is a linear function of m [19], of the form $Y_m = (1 + \alpha_n)m$, with the coefficient α_n introduced earlier, the tangent is also correct at $\mathbf{m} = -\mathbf{m}^*$. In that case, the previous derivation applies directly in terms of the effective field $B_{eff} = (1+\alpha_n)B$, so that the bound of Eq. (17) becomes:

$$\frac{\mathrm{Var}(\mathbf{m})}{\langle m\rangle} \leq \frac{k_B T}{n B_{eff}}. \tag{20}$$

The bounds for *global* and *local* fluctuations, namely Eqs. (17)-(20), represent our second main result. These bounds are confirmed by direct simulations of the 2D Ising model shown in fig. 6.

To conclude, we have studied thermodynamic constraints on equilibrium fluctuations, which are imposed by isometric fluctuation theorems. These fluctuation theorems have been derived for a large class of systems with discrete or continuous broken symmetries [10, 11]. Interestingly, corrections to the fluctuation theorems due to coarse-graining only occurs at intermediate scales, which are neither small nor large with respect to the size of the full system. These corrections arise due to the statistical dependence between the sub-part and its complement, which can be measured by their mutual information [23].

We have also derived general thermodynamic bounds on the variance of equilibrium fluctuations of a *global* and *local* order parameter. The relations for a *global* order parameter are analogous to relations derived recently in the non-equilibrium case [20, 21]. Such bounds on the equilibrium order parameter fluctuations can be tested experimentally more easily than their large deviation counterparts (whether at equilibrium or out of equilibrium). The extension for *local* fluctuations appears promising to infer the value of the symmetry breaking field or the relative size of the observation window with respect to the full system.

ACKNOWLEDGMENTS

The authors thank P. Gaspard for a previous collaboration on which this work was built.

[1] D. J. Evans, E. G. D. Cohen, and G. P. Morriss, Phys. Rev. Lett. **71**, 2401 (1993).
[2] G. Gallavotti and E. G. D. Cohen, Phys. Rev. Lett. **74**, 2694 (1995).
[3] G. E. Crooks, Phys. Rev. E **61**, 2361 (2000).
[4] J. Kurchan, arXiv:0901.1271 (2009).
[5] C. Jarzynski, Annu. Rev. Condens. Matter Phys. Annual Review of Condensed Matter Physics, **2**, 329 (2011).
[6] U. Seifert, Rep. Prog. Phys. **75**, 126001 (2012).
[7] N. Goldenfeld, *Lectures On Phase Transitions And The Renormalization Group*, edited by F. in Physics (Perseus Books Publishing, L.L.C., 1992).
[8] P. Gaspard, J. Stat. Mech. **2012**, P08021 (2012).
[9] P. I. Hurtado, C. Pérez-Espigares, J. J. del Pozo, and P. L. Garrido, Proc. Natl. Acad. Sci. U.S.A. **108**, 7704 (2011).
[10] D. Lacoste and P. Gaspard, Phys. Rev. Lett. **113**, 240602 (2014).
[11] D. Lacoste and P. Gaspard, J. Stat. Mech. (2015).
[12] R. García-García, S. Lahiri, and D. Lacoste, Phys. Rev. E **93**, 032103 (2016).
[13] A. Alemany, M. Ribezzi-Crivellari, and F. Ritort, New Journal of Physics **17**, 075009 (2015).
[14] G. Michel and D. J. Searles, Phys. Rev. Lett. **110**, 260602 (2013).
[15] M. Esposito, Phys. Rev. E **85**, 041125 (2012).
[16] G. Bulnes Cuetara, M. Esposito, and P. Gaspard, Phys. Rev. B **84**, 165114 (2011).
[17] D. Lacoste and K. Mallick, Phys. Rev. E **80**, 021923 (2009).
[18] S. Rahav and C. Jarzynski, J. Stat. Mech.: Theory , P09012 (2007).
[19] See Supplemental Material at http://link.aps.org/ for details on the derivation of the bound for the fluctuations of the order parameter.
[20] A. C. Barato and U. Seifert, Phys. Rev. Lett. **114**, 158101 (2015).
[21] T. Gingrich, arXiv:1512.02212v3 (2016).
[22] H. Touchette, Phys. Rep. **478**, 1 (2009).
[23] O. Cohen, V. Rittenberg, and T. Sadhu, J. Phys. A-Math. Gen. **48**, 055002 (2015).