son for being contented with this approximation. **We** may now make uſe of the formula expreſſing the velocity for ſolving the chief problems in this part of the ſeaman’s taſk.

And first let it be required to determine the beſt poſition of the ſail for ſtanding on a given courſe *ab,* when CE the direction and velocity of the wind, and its angle with the courſe WCF, are given. This problem has exerciſed the talents of the mathematicians ever ſince the days oſ Newton. In the article Pneumatics we give the ſolution of one very nearly related to it, name­ly, to determine the poſition of the ſail which would produce the greateſt impulſe in the direction of the courſe. The ſolution was to place the yard CD in ſuch a poſition that the tangent of the angle FCD may be one half of the tangent of the angle DCW. This will indeed be the beſt poſition of the ſail for beginning the motion; but as ſoon as the ſhip begins to move in the direction CF, the effective impulſe of the wind is diminiſhed, and alſo its inclination to the ſail. The angle DC w diminiſhes: continually as the ſhip accelerates; for CF is now accompanied by its equal *e* E, and by an angle ECe or WC*w.* CF increaſes, and the impulſe on the sail diminiſhes, till an equilibrium obtains between the reſiſtance of the water and the im­pulſe of the wind. The impulſe is now meaſured by Ce2 × sin.*2 e*CD inſtead of CE2 × sin.2 ECD, that is, by EG2 inſtead of Eg2.

This introduction of the relative motion oſ the wind renders the actual ſolution of the problem extremely difficult. It is very eaſily expreſſed geometrically: Divide the angle *w*CF in ſuch a manner that the tan­gent of DCF may be half of the tangent of DCw, and the problem may be conſtructed geometrically as fol­lows.

Let WCF (fig. 7.) be the angle between the ſail and courſe. Round the centre C deſcribe the circle WDFY; produce WC to Q, ſo that CQ = 1/3W C, and draw QY parallel to CF cutting the circle in Y; biſect the arch WY in D, and draw DC. DC is the proper poſition of the yard.

Draw the chord WY, cutting CD in V and CF in T; draw the tangent PD cutting CF in S and CY in R.

It is evident that WY, PR, are both perpendicular to CD, and are biſected in V and D; therefore (by reaſon of the parallels QY, CF) 4 : 3 = QW : CW, = YW : TW, = RP : SP. Therefore PD : PS = 2 : 3, and PD : DS = 2 : 1.  *E. D.* But this diviſion cannot be made to the beſt advantage till the ſhip has attained its greateſt velocity, and the angle wCF has been produced.

We muſt conſider all the three angles, a, *b,* and *x* as variable in the equation which expreiſes the value of *v,* and we muſt make the fluxion of this equation = o; then, by means of the equation B = A· cotan. *b,* w,e muſt obtain the value of *b* and of *b* in terms of *x* and x. With reſpect to a, observe, that if we make the angle WCF = p, we have p = *a+ b + x;* and p being a con­stant quantity, we have *a + b + x = 0.* Subſtituting for a, *b, a,* and *b,* their values in terms of *x* and *x,* in the fluxionary equation = 0*,* we readily obtain *x,* and then *a* and *b,* which ſolves the problem.

Let it be required, in the next place, to determine the courſe and the trim of the sails moſt proper for ply­ing to windward.

In fig. 6. draw FP perpendicular to WC. CF is the motion of the ſhip; but it is only by the motion CP that ſhe gains to windward. Now CP is = CF × coſin. WGF, or *ν* coſin. (a+ b + x). This muſt be ren­dered a maximum, as follows.

By means of the equation which expreſſes the value of *v* and the equation B = A · cotan. *b,* we exterminate the quantities *v* and *b;* we then take the fluxion of the quantity into which the expreſſion *v* · coſ. ( a + b + x) is changed by this operation. Making this fluxion = 0, we get the equation which muſt ſolve the problem. This equation will contain the two variable quantities *a* and x with their fluxions; then make the coefficient of x equal to o*,* alſo the coefficient of *a* equal to o*.* This will give two equations which will determine *a* and *x,* and from this we get *b = p — a —x.*

Should it be required, in the third place, to find the beſt courſe and trim of the ſails for getting away from a given line of coaſt CM (fig. 6.), the proceſs perfectly reſembles this laſt, which is in fact getting away from a line of coaſt which makes a right angle with the wind. Therefore, in place of the angle WCF, we muſt substitute the angle WCM c± WCF. Call this angle *e.* We muſt make *v* · coſ. *(e = a = b =x)* a maximum. The analytical proceſs is the ſame as the former, only e is here a conſtant quantity.

Theſe are the three principal problems which can be ſolved by means of the knowledge that we have obtain­ed of the motion of the ſhip when impelled by an ob­lique ſail, and therefore making leeway; and they may be conſidered as an abſtract of this part of M. Bouguer’s work. We have only pointed out the proceſs for this ſolution, and have even omitted ſome things taken notice of by M. Bezout in his very elegant compendium. Our reaſons will appear as we go on. The learned reader will readily ſee the extreme difficulty of the ſubject, and the immenſe calculations which are neceſſary even in the ſimpleſt cases, and will grant that it is out of the power of any but an expert analyſt to derive any uſe from them; but the mathematician can calculate tables for the uſe of the practical ſeaman. Thus he can calculate the best poſition of the ſails for advancing in a courſe 900 from the wind, and the velocity in that courſe; then for 85, 80⁰, 75⁰, &c. M. Bouguer has given a table of this kind; but to avoid the immenſe difficulty of the proceſs, he has adapted it to the apparent direction of the wind. We have inſerted a few of his numbers, ſuited to ſuch caſes as can be of ſervice, namely, when all the ſails draw, or none stand in the way of others. Co­lumn 1st is the apparent angle of the wind and courſe; column 2d is the correſponding angle of the ſails and keel; and column, 3d is the apparent angle of the ſails and wind.

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| --- | --- | --- |
| 1 | 2 | 3 |
| wCF | DCB | wCD |
| 1043⁰ 53' | 42⁰ 30' | *61*⁰ *23'* |
| 99 13 | 40 — | 59 13 |
| 94 25 | 37 30 | 56 55 |
| 89 28 | 35 — | 54 28 |
| 84 23 | 32 30 | 51 53 |
| 79 06 | 30 — | 49 06 |
| 73 39 | 27 30 | 46 09 |
| 68 — | 25 — | 43 — |