Fig. 5. n⁰ 2. A is a perſpective view of a three ſided beam projecting horizontally from a wall, and loaded with a weight at B just sufficient to break it. DABC is a vertical plane through its higheſt point D, in the direction of its length. a D *a* is another vertical ſection perpendicular to AB. The piece being suppoſed of insuperable strength everywhere ex­cept in the ſection *a* D *a,* and the coheſion being alſo ſupposed insuperable along the line a A a, it can break nowhere but in this ſection, and by turning round a A a as round a hinge. Make D *d* equal to AD, and let D *d* repreſent the absolute coheſion of the fibre at D, which abſolute coheſion we expreſſed by the ſymbol f*.* Let a plane a *d a* be made to pass through *a a* and *d,* and let *d' d' d'* be another croſs ſection. It is plain that the priſmatic ſolid contained be­tween the two sections *a D a* and *a'da'* will repreſent the full coheſion of the whole ſection of fracture ; for we may conceive this prism as made up of lines ſuch as F *f,* equal and parallel to D d, repreſenting the abſolute coheſion of each particle ſuch as F. The pyramidal ſolid *d* D *a a,* cut off by the plane *da a,* will repreſent the coheſions actua*lly exerted* by the different fibres in the inſtant of fracture. For take any point E in the ſurface of fracture, and draw Ee parallel to AB, meeting the plane *ada* in *e,* and let eA E be a vertical plane. It is evident that D *d* is to E *e* as AD to AE; and therefore ſince the forces exerted by the different fibres are as their extenſion, and their exten­sion as their diſtances from the axis of fracture) E *e* will re­preſent the force actually exerted by the fibre in E, while D is exerting its full force D *d.* In like manner, the plane F F f f expreſſes the coheſion exerted by all the fibres in the line F F, and ſo on through the whole ſurface. Therefore the pyramid *d a a* D expreſſes the accumulated exertion of the whole ſurface of fracture.

Farther, suppoſe the beam to be held perpendicular to the horizon with the end B uppermoſt, and that the weight of the priſm contained between the two ſections *a* D *a* and *a' d a'* (now horizontal) is just able to overcome the full co­heſion of the ſection of fracture. The weight of the pyra­mid d D *a a* will alſo be juſt able to overcome the coheſions actu*ally exerted* by the different fibres in the inſtant of frac­ture, becauſe the weight of each fibre, ſuch as E *e,* is juſt ſuperior to the coheſion actually exerted at E.

Let o be the centre of gravity of the pyramidal ſolid, and draw *o* O perpendicular to the plane *a* D *a.* The whole weight of the ſolid *d* D *a a* may be conceived as accumula­ted in the point o, and as acting on the point O, and it will have the ſame tendency to ſeparate the two cohering ſurfaces as when each fibre is hanging by its respective point. For this reaſon the point O may be called the *centre of actual effort* of the unequal forces of coheſion. The momentum there­fore, or energy by which the cohering ſurfaces are ſeparated, will be properly meaſured by the weight of the ſolid d D *a a* multiplied by OA ; and this product is equal to the product of the weight *f* multiplied by BA, or by l. Thus ſuppoſe that the coheſion along the line AD only is conſidered. The whole coheſion will be repreſented by a tri­angle. A D *d* D d repreſents *f,* and AD is *d,* and AD is *x.* Therefore A D *d* is 1/2f*d.* The centre of gravity o of the triangle A D J is in the interſection of a line drawn from A to the middle of D *d* with a line drawn from *d* to the middle of AD ; and therefore the line *o O* will make AO — 2/3 of A D. Therefore the actual momentum of coheſion is f × 1/2*d × 2/3d, = f × d × 1/3d, = fd × 1/3d,* or equal to the abſolute coheſion acting by means of the lever d/3. If the ſection of fracture is a rectangle, as in a common joiſt, whoſe breadth *a a* is = *b,* it is plain that all the vertical lines will be equal to AD, and their coheſions will be repreſented by triangles like A D *d ;* and the whole actual coheſion will be repreſented by a wedge whoſe baſes are vertical planes, and which is equal to half of the parallelopiped AD ×D *d × a a,* and will therefore be = 1/2fbd and the diſtance AO of its centre of gravity from the horizontal line A A' will be 2/3 of A D, The momentum of coheſion of a joiſt will therefore be 1/2f*bd* × 2/3*d, or fbd* × *1/3d,* as we have determined in the other way.

The beam repreſented in the figure is a triangular priſm. The pyramid D *a a d* is 1/3 of the priſm *a a* D *d a' a'.* If we make s repreſent the ſurface of the triangle *a* D *a,* the pyra­mid is 1/3 of f*s.* The diſtance AO of its centre of gravity from the horizontal line A A' is 1/2 of A D, or 1/2 *d.* There­fore the momentum of actual coheſion is 1/3fs *× 1/2d, = fs* × 1/6*d;* that is, it is the ſame as if the full coheſion of all, the fibres were accumulated at a point I whoſe diſtance from A is 1/6th of AD or d, or (that we may ſee its value in every point of view) it is 1/6th of the momentum of the full coheſion of all the fibres when accumulated at the point D, or acting at the diſtance *d* = A D.

This is a very convenient way of conceiving the momen­tum of actual coheſion, by comparing it with the mo­mentum of abſolute coheſion applied at the diſtance AD from the axis of fracture. The momentum of the abſo­lute coheſion applied at D is to the momentum of actual coheſion in the inſtant of fracture as AD to AI. There­fore the length of AI, or its proportion to AD, is a sort of index of the strength of the beam. We ſhall call it the Index, and expreſs it by the ſymbol *i.*

Its value is eaſily obtained. The product of the abſolute coheſion by AI must be equal to that of the actual coheſion by AO. Therefore ſay, “ as the priſmatic ſolid *a a* D *d d a'* is to the pyramidal ſolid *a a* D *d,* so is AO to A I.” We are aſſiſted in this determination by a very con­venient circumſtance. In this hypotheſis of the actual co­heſions being as the diſtances of the fibres from A, the point O is the centre of oſcillation or percuſſion of the ſur­face D *a a* turning round the axis *a a:* for the momentum of coheſion of the line FF is FF × Ff × EA = FF×E A2, becauſe Ff is equal to E A. Now AO, by the nature of the centre of gravity, is equal to the ſum of all theſe mo­menta divided by the pyramid a *a* D *d* ; that is, by the ſum of all the FF × Ff that is, by the ſum of all the FF×E A. Therefore AO = (ſum of FF × E A2)/(sum of FF × EA), which is just the value of the diſtance of the centre of percuſſion of the triangle *a a D* from A: (See Rotation). Moreover, if G be the centre of gravity of the triangle *a* D *a,* we ſhall have D A to G A as the abſolute coheſion to the ſum of the coheſions actually exerted in the inſtant of fracture ; for, by the nature of this centre of gravity, A G is equal to (ſum of FF × EA), and the sum of FF × AG is equal to the ſum of FF × EA. But the ſum of all the lines F F is the triangle *a* D *a,* and the ſum of all the F F × E A is the ſum of all the rectangles F ff that is, the pyramid *d* D *a a.* Therefore a priſm whoſe baſe is the triangle *a* D *a,* and whoſe height is AG, is equal to the pyramid, or will expreſs the ſum of the actual coheſions ; and a priſm, whoſe baſe is the ſame triangle, and whoſe height is D *d* or D *a,* expreſſes the abſolute coheſion. Therefore D A is to G A as the abſolute coheſion to the ſum of the actual coheſions.

Therefore we have DA:GA = OA:IA.

Therefore, whatever be the form of the beam, that is, whatever be the figure of its ſection, find the centre of oſcillation O, and the centre of gravity G of this ſection.