outer fibres is proportional to the curvature ; for, becauſe the curves formed by the inner and outer ſides of the beam are ſimilar, the circumferences are as the radii, and the ra­dius of the inner circle is to the difference of the radii as the length of the inner circumference is to the difference of the circumferences. The difference of the radii is the depth of the beam, the difference of the circumferences is the extenſion of the outer fibres, and the inner circumference is ſuppoſed to be the primitive length of the beam. Now the ſecond and third quantities of the above analogy, viz. the depth and length of the beam, are constant quantities, as is alſo their product. Therefore the product of the inner radius and the extenſion of the outer fibre is alſo a conſtant quan­tity, and the whole extenſion of the outer fibre is inverſely as the radius of curvature, or is directly as the curvature of the beam.

The mathematical reader will readily see, that into what­ever curve the elaſtic bar is bent, the whole extenſion of the outer fibre is equal to the length of a ſimilar curve, having the ſame proportion to the thickneſs of the beam that the length of the beam has to the radius of curvature.

Now let ADCB (fig. 5. n⁰ 3.) be ſuch a rod, of uniform breadth and thickneſs, firmly fixed in a vertical position, and bent into a curve AEFB by a weight W ſuſpended at B, and of ſuch magnitude that the extremity B has its tangent perpendicular to the action of the weight, or parallel to the horizon. Suppoſe too that the extenſions are proportional to the extending forces. From any two points E and F draw the horizontal ordinates EG, FH. It is evident that the exterior fibres of the ſections E *e* and Ff are ſtretched by forces which are in the proportion of EG to FH (theſe being the long arms of the levers, and the equal thickneſſes E*e,* Ff being the ſhort arms). Therefore (by the hypotheſis) their extenſions are in the ſame proportion. But becauſe the extenſions are proportional to ſome ſimilar functions of the diſtance from the axes of fracture E and F, the extenſion of any fibre in the ſection E *e* is to the contem­poraneous extenſion of the ſimilarly ſituated fibre in the ſection Ff, as the extenſion of the exterior fibre in the ſection E*e* is to the extenſion of the exterior fibre in the ſection Ff: therefore the whole extenſion of E*e* is to the whole extenſion of Ff as EG to FH, and EG is to FH as the curvature in E to the curvature in F.

Here let it be remarked, that this proportionality of the curvature to the extenſion of the fibres is not limited to the hypotheſis of the proportionality of the extenſions to the ex­tending forces. It follows from the extenſion in the dif­ferent ſections being as ſome ſimilar function of the diſtance from the axis of fracture ; an assumption which cannot be refuſed.

This then is the fundamental property of the elaſtic curve, from which its equation, or relation between the abſciſſa and ordinate, may be deduced in the uſual forms, and all its other geometrical properties. Theſe are foreign to our purpoſe ; and we ſhall notice only ſuch properties as have an immediate relation to the ſtrain and ſtrength of the dif­ferent parts of a flexible body, and which in particular ſerve to explain ſome difficulties in the valuable experiments of Mr Buffon on the Strength of Beams.

We obſerve, in the firſt place, that the elaſtic curve cannot be a circle, but is gradually more incurvated as it recedes from the point of application B of the ſtraining forces. At B it has no curvature ; and if the bar were extended be­yond B there would be no curvature there. In like manner, when a beam is ſupported at the ends and loaded in the mid­dle, the curvature is greateſt in the middle ; but at the props, or beyond them, if the beam extend farther, there is no curvature. Therefore when a beam projecting 20 feet from a wall is bent to a certain curvature at the wall by a weight ſuſpended at the end, and a beam of the ſame ſize projecting 20 feet is bent to the very ſame curvature at the wall by a greater weight at 10 feet diſtance, the figure and the me­chanical ſtate of the beam in the vicinity of the wall is dif­ferent in theſe two caſes, though the curvature at the very wall is the ſame in both. In the firſt caſe every part of the beam is incurvated ; in the ſecond, all beyond the 10 feet is without curvature. In the firſt experiment the curva­ture at the diſtance of five feet from the wall is 3/4ths of the curvature at the wall ; in the ſecond, the curvature at the ſame place is but 1/2 of that at the wall. This muſt weaken the long beam in this whole interval of five feet, becauſe the greater curvature is the reſult of a greater extenſion of the fibres.

In the next place, we may remark, that there is a certain determinate curvature for every beam which cannot be ex­ceeded without breaking it ; for there is a certain ſeparation of two adjoining particles that puts an end to their co­heſion. A fibre can therefore be extended only a certain proportion of its length. The ultimate extenſion of the outer fibres muſt bear a certain determinate proportion to its length, and this proportion is the ſame with that of the thickneſs (or what we have hitherto called the depth) to the radius of ultimate curvature, which is therefore deter­minate.

A beam of uniform breadth and depth is therefore most incuryated where the ſtrain is greateſt, and will break in the moſt incurvated part. But by changing its form, so as to make the ſtrength of its different ſections in the ratio of the ſtrain, it is evident that the curvature may be the ſame throughout, or may be made to vary according to any law. This is a remark worthy of the attention of the watchmaker. The moſt delicate problem in practical me­chanics is ſo to taper the balance-ſpring of a watch that its wide and narrow vibrations may be isochronous. Hooke’s principle *ut tenſio sic vis* is not sufficient when we take the *inertia* and motion of the ſpring itſelf into the account. The figure into which it bends and unbends has alſo an influence. Our readers will take notice that the artiſt aims at an ac­curacy which will not admit an error of 1/86400th, and that Harriſon and Arnold have actually attained it in ſeveral inſtances. The taper of a ſpring is at preſent a noſtrum in the hands of each artiſt, and he is careful not to impart his ſecret.

Again, ſince the depth of the beam is thus proportional to the radius of ultimate curvature, this ultimate or break­ing curvature is inverſely as the depth. It may be expressed by 1/d.

When a weight is hung on the end of a priſmatic beam, the curvature is nearly as the weight and the length directly, and as the breadth and the cube of the depth inverſely ; for the ſtrength is = f(bd2/3l). Let us ſuppoſe that this produces the ultimate curvature 1/d. Now let the beam be loaded with a ſmaller weight w, and let the curvature produced be C, we have this analogy f(bd2/3l) : w = 1/d : C, and C = 3lw/fbd3. evident that this is alſo true of a beam ſupported at the ends and loaded between the props ; and we ſee how to determine the curvature in its different parts, whether ariſing from the load, or from its own weight, or from both.

When beam is thus loaded at the end or middle, the