It deferves to be noticed, that although the relative ſtrength oſ a prismatic ſolid is extremely different in the three hypotheſes now considered, yet the proportional ſtrengths of different pieces follow the ſame ratio ; namely, the direct: ratio of the breadth, the direct ratio of the ſquare of the depth, and the inverſe ratio of the length. In the firſt hypotheſis (of equal forces) the ſtrength of a rectangular beam was fbd2/2l ; in the ſecond (of attractive forces proportional to the extenſions) it was fbd2/3l ; and in the third (equal attractions and repulſions proportional to the extenſions and compreſſions) it was fbd2/6l, or more generally fbd2/ml, where *m* expreſſes the unknown proportion between the attractions and repulſions correſponding to an equal extension and compreſſion.

Hence we derive a piece of uſeful information, which is confirmed by unexcepted experience, that the ſtrength of a piece depends chiefly on its depth, that is, on that dimenſion which is in the direction of the ſtrain. A bar of timber of one inch in breadth and two inches in depth is four times as ſtrong as a bar of only one inch deep, and it is twice as ſtrong as a bar two inches broad and one deep ; that is, a joiſt or lever is always ſtrongeſt when laid on its edge.

There is therefore a choice in the manner in which the coheſion is oppoſed to the ſtrain. The general aim muſt be to put the centre of effort I as far from the fulcrum or the neutral point A as poſſible, ſo as to give the greateſt energy or momentum to the coheſion. Thus if a triangular bar projecting from a wall is loaded with a weight at its extre­mity, it will bear thrice as much when one of the ſides is uppermoſt as when it is undermoſt. The bar of fig. 5. n⁰ 2. would be three times as ſtrong if the side AB were uppermoſt and the edge DC undermoſt.

Hence it follows that the ſtrongeſt joiſt that can be cut out of a round tree is not the one which has the greateſt quantity of timber in it, but ſuch that the product of its breadth by the ſquare of its depth ſhall be the greateſt poſ­ſible. Let ABCD (fig. 7.) be the lection of this joiſt inſcribed in the circle, AB being the breadth and AD the depth. Since it is a rectangular ſection, the diagonal BD is a diameter of the circle, and BAD is a right angled tri­angle. Let BD be called *a,* and BA be called *x* ; then AD is = Vz*az—x2.* Now we muſt have AB × AD2, or x × *a2 —* x2, or *a2x — x3, a* maximum. Its fluxion a2x*—3x2x* muſt be made = o, or *a2 =* 3x2, or *x2 = a2/3.* If therefore we make DE = 1/3DB, and draw EC perpen­dicular to BD, it will cut the circumference in the point C, which determines the depth BC and the breadth CD.

Becauſe BD : BC = CD : CE, we have the area of the ſection BC × CD = BD × CE. Therefore the different sec­tions having the ſame diagonal BD are proportional to their heights CE. Therefore the ſection BCDA is leſs than the ſection B*c*Da, whoſe four ſides are equal. The joiſt ſo ſhaped, therefore, is both ſtronger, lighter, and cheaper.

The ſtrength of ABCD is to that of *a*B*e*D as 10,000 to 9186, and the weight and expence as 10,000 to 10,607; ſo that ABCD is preferable to *a* B *c* D in the proportion of 10,607 to 9186, or nearly 115 to 100.

From the ſame principles it follows that a hollow tube is ſtronger than a ſolid rod containing the same quantity of matter. Let fig. 8. repreſent the ſection of a cylindric tube, of which AF and BE are the exterior and interior diameters and C the centre. Draw BD perpendicular to BC, and join DC. Then, becauſe BD2 = CD2 — CB2, BD is the radius of a circle containing the ſame quantity of matter with the ring. If we eſtimate the ſtrength by the firſt hypotheſis, it is evident that the ſtrength of the tube will be to that of the ſolid cylinder, whoſe radius is BD, as BD2 × AC to BD2 × BD ; that is, as AC to BD : for BD2 expreſſes the coheſion of the ring or the circle, and AC and BD are equal to the distances of the centres of ef­fort (the ſame with the centres of gravity) of the ring and circle from the axis of fracture.

The proportion of theſe ſtrengths will be different in the other hypotheſes, and is not eaſily expreſſed by a general formula ; but in both it is ſtill more in favour of the ring or hollow tube.

The following very simple ſolution will be readily underſtood by the intelligent reader. Let O be the centre of oſcillation of the exterior circle, *o* the centre of oſcillation of the inner circle, and w the centre of oſcillation of the ring included between them. Let M be the quantity of ſurface of the exterior circle, *m* that of the inner circle, and *μ* that of the ring. We have (Fw = M × FO - m × FO)/μ, = (5FC2 + EC2)/4FC, and the ſtrength of the ring = (fμ × FW)/2, and the ſtrength of the ſame quantity of matter in the form of a ſolid cylinder is fμ × 5/8BD ; ſo that the ſtrength of the ring is to that of the ſolid rod of equal weight as F to 5/4BD, or nearly as FC to BD. This will eaſily appear by recollecting that FO is = (sum of p × r2)/(m × FC) (see Rotation), and that the momentum of coheſion is (fm × FC × Fa)/(2FC) = (fm × Fo)/2 for the inner circle, &c.

Emerſon has given a very inaccurate approximation to this value in his *Mechanics,* 4to.

This property of hollow tubes is accompanied alſo with greater ſtiffneſs ; and the ſuperiority in ſtrength and ſtiffness is ſo much the greater as the ſurrounding ſhell is thinner in proportion to its diameter.

Here we see the admirable wiſdom of the Author of nature in forming the bones of animal limbs hollow. The bones of the arms and legs have to perform the office of le­vers, and are thus oppoſed to very great tranſverſe ſtrains. By this form they become incomparably ſtronger and ſtiffer, and give more room for the inſertion of muſcles, while they are lighter and therefore more agile ; and the ſame Wisdom has made uſe of this hollow for other valuable purposes of the animal economy. In like manner the quills in the wings of birds acquire by their thinneſs the very great ſtrength which is necessary, while they are ſo light as to give ſufficient buoyancy to the animal in the rare medium in which it mult live and fly about. The ſtalks of many plants, ſuch as all the graſſes, and many reeds, are in like manner hollow, and. thus posseſs an extraordinary ſtrength. Our beſt engineers now begin to imitate nature by making many parts of their machines hollow, ſuch as their axles of cast iron, &c. ; and the ingenious Mr Ramſden now makes the axes and framings of his great aſtronomical inſtruments in the ſame manner.

In the ſupposition of homogeneous texture, it is plain that the fracture happens as ſoon as the particles at D are ſeparated beyond their utmoſt limit of coheſion. This is a determined quantity, and the piece bends till this degree of extenſion is produced in the outermoſt fibre. It follows, that the ſmaller we ſuppoſe the diſtance between A and D,