*a*AC is two-thirds of the cylinder, becauſe the cone gene­rated by *c*C*a* is one-third of it.

We may now ſuppoſe the cylinder twiſted till the parti­cles in the external circumference lose their coheſion. There can be no doubt that it will now be wrenched aſunder, all the inner circles yielding in ſucceſſion. Thus we obtain one uſeful information, viz. that a body of homogeneous texture refills a sim*ple twist* with two-thirds of the force with which it resists an attempt to force one part laterally from the other, or with one-third part of the force which will cut it aſunder by a ſquare-edged tool. For to drive a ſquare- edged tool through a piece of lead, for inſtance, is the ſame as forcing a piece of the lead as thick as the tool laterally away from the two pieces on each side of the tool. Experiments of this kind do not ſeem difficult, and they would give us very uſeful information.

When two cylinders AHK and BNO are wrenched asunder, we muſt conclude that the external particles of each are just put beyond their limits of coheſion, are equally ex­tended, and are exerting equal forces. Hence it follows, that in the inſtant of fracture the ſum total of the forces ac­tually exerted are as the ſquares of the diameters.

For drawing the diagonal C , it is plain that Er, = Aa, expreſſes the distension of the circumference ELM, and that the ſolid generated by the triangle CE*e* expreſſes the co­heſion exerted by the ſurface of the circle ELM, when the particles in the circumference ſuffer the extenſion E*e* equal to Aa. Now the solids generated by CA*a* and CE*e* be­ing respectively two thirds of the correſponding cylinders, are as the ſquares of the diameters.

Having thus aſcertained the real ſtrength of the ſection, and its relation to its abſolute lateral ſtrength, let us examine its ſtrength relative to the external force employed to break it. This examination is very simple in the case un­der conſideration. The straining force muſt act by ſome le­ver, and the coheſion muſt oppoſe it by acting on ſome other lever. The centre of the ſection may be the neutral point, whoſe position is not diſturbed.

Let F be the force exerted laterally by an exterior par­ticle. Let *a* be the radius of the cylinder, and *χ* the inde­terminate diſtance of any circumference, and *x* the indefinitely ſmall interval between the concentric arches ; that is, let *X* be the breadth of a ring and *x* its radius. The for­ces being as the extenſions, and the extenſions as the diſtances from the axis, the coheſion actually exerted at any part of any ring will be fxx/a. The force exerted by the whole ring (being as the circumference or as the radius) f(x2x)/a. The momentum of coheſion of a ring, be­ing as the force multiplied by its lever, will be f(x3x)/a. The accumulated momentum will be the ſum or fluent of f(x3x)/a; that is, when *x = a,* it will be 1/4f(a4/a), = 1/4fa3.

Hence we learn that the ſtrength of an axle, by which it reſiſts being wrenched aſunder by a force acting at a given diſtance from the axis, is as the cube of its diameter.

But farther, 1/4f*a*3 is *= fa*2 × 1/4*a.* Now fa2 repreſents the full lateral coheſion of the ſection. The momen­tum thereſore is the ſame as if the full lateral coheſion were accumulated at a point diſtant from the axis by 1/4th of the radius or ⅛th of the diameter of the cylinder.

Therefore let F be the number of pounds which meaſures the lateral coheſion of a circular inch, *d* the diameter of the cylinder in inches, and l the length of the lever by which the ſtraining force p is ſuppoſed to act, we ſhall have F × 1/8d3 = pl, and F(d3/8l) = p. We see in general that the ſtrength of an axle, by which it reſiſts being wrenched aſunder by twiſting, is as the cube of its diameter.

We ſee alſo that the internal parts are not acting ſo powerfully as the external. If a hole be bored out of the axle of half its diameter, the ſtrength is diminiſhed only 1/8th, while the quantity of matter is diminiſhed 1/4th. Therefore hollow axles are ſtronger than ſolid ones containing the ſame quantity of matter. Thus let the diameter be 5 and that of the hollow 4 : then the diameter of another ſolid cylinder having the ſame quantity of matter with the tube is 3. The ſtrength of the ſolid cylinder of the diameter 5 may be expreſſed by *53 or* 125. Of this the internal part (of the diameter 4) exerts 64 ; therefore the ſtrength of the tube is 125 — 64, = 61. But the ſtrength of the ſolid axle of the ſame quantity of matter and diameter 3 is 33, or 27, which is not half of that of the tube.

Engineers, therefore, have of late introduced this improvement in their machines, and the axles of caſt iron are all made hollow when their size will admit it. They have the additional advantage of being much ſtiffer, and of affording much better fixure for the flanches, which are uſed for connecting them with the wheels or levers by which they are turned and ſtrained. The ſuperiority of ſtrength of hollow tubes over ſolid cylinders is much greater in this kind of ſtrain than in the former or tranſverſe. In this laſt case the strengh of this tube would be to that of the ſolid cylinder of equal weight as 61 to 321/2. nearly.

The apparatus which we mentioned on a former occaſion for trying the lateral ſtrength of a ſquare inch of solid mat­ter, enabled us to try this theory of twiſt with all desirable ac­curacy. The bar which hung down from the pin in the for­mer trials was now placed in a horizontal poſition, and loaded with a weight at the extremity. Thus it acted as a powerful lever, and enabled us to wrench aſunder ſpecimens of the ſtrongeſt materials. We found the reſults perfectly con­formable to the theory, in as far as it determined the pro­portional ſtrength of different sizes and forms : but we found the ratio of the reſistance to twiſting to the ſimple, lateral resistance considerably different; and it was ſome time before we diſcovered the cauſe.

We had here taken the ſimpleſt view that is poſſible of the action, of coheſion in reſiſting a twill. It is frequently exerted in a very different way. When, for inſtance, an iron axle is joined to a wooden one by being driven into one end of it, the extenſions of the different circles of par­ticles are in a very different proportion. A little con­ſideration will ſhow that the particles in immedate contact with the iron axle are in a ſtate of violent extenſion ; so are the particles of the exterior ſurface of the wooden part, and the intermediate parts are leſs ſtrained. It is almoſt impoſſible to aſſign the exact proportion of the coheſive forces exerted in the different parts. Numberleſs cases can be pointed out where parts of the axle are in a ſtate of compreſſion, and where it is ſtill more difficult to determine the ſtate of the other particles. We must content ourſelves with the deductions made from this ſimple case, which is fortunately the moſt common. In the experiments just now mentioned the centre of the circle is by no means the neu­tral point, and it is very difficult to aſcertain its place : but when this conſideration occurred to us, we eaſily freed the ex­periments from this uncertainty, by extending the lever to both sides, and by means of a pulley applied equal force