*Cor.* **2.** The lateral aberration FG is = AV3/2CV2. For

FG : Ff = AP : Pf, = AV : 1/2CV nearly, and therefore FG = AV3/4CV × *2/CV =* AV3/2CV2.

2. I*n Refractions.*

Let AVB (fig. 4. A or B) be a ſpherical surface ſeparating two refracting ſubſtances, C the centre, V the vertex, AV the ſemi-aperture, AP its sine, PV its verſed sine, and F the focus of parallel rays infinitely near to the axis. Let the extreme ray *a* A, parallel to the axis, be refracted into AG, croſſing CF in f*,* which is therefore the focus of ex­treme parallel rays.

*The rectangle of the sine of incidence, by the difference of the sines of incidence and refraction, is to the ſquare of the sine of re­fraction, as the verſed sine of the ſemi-aperture is to the longitudi­nal abberration of the extreme rays.*

Call the sine of incidence *i,* the sine of refraction r, and their difference *d.*

Join CA, and about the centre *f* deſcribe the arch AD.

The angle ACV is equal to the angle of incidence, and CA*f* is the angle of refraction. Then, ſince the sine of incidence is to the sine of refraction as VF to CF, or as Af to *Cf,* that is, as Df to Cf, we have by converſion: CF: FV = Cf : fD., CF : CV = Cf : CD.

altern. conver: CF — C*f* : CV — CD = CF : CV.

or: Ff : VD = CF : CV, = *r : d.*

Now PV = AP2/(CP + CV), = AP2/2CV nearly, and PD = AP2/(fP + fV) = AP2/2fV nearly, = AP2/2FV nearly. Therefore PV : PD = FV : CV, and DV : PV = CF : FV nearly.

We had above: F*f* : VD = *r : d;*

and now: VD : PV = CF : FV, = r : i;

therefore: Ef : PV = *r2 : di,*

and: Ff × PV. ED*.*

The abberration will be different according as the refrac­tion is made towards or from the perpendicular; that is, ac­cording as *r* is leſs or greater than *i.* They are in the ratio of r2/di to i2/dr, or of *r3* to *i3.* The abberration therefore is always much diminiſhed when the refraction is made from a rare into a denſe medium. The proportion of the sines for air and glaſs is nearly that of 3 to 2. When the light is refracted into the glaſs, the abberration is nearly 4/3 of PV ; and when the light passes out of glaſs into air, it is about 9/2 of PV.

*Cor.* 1. Ef = r2/di × AP2/2CV nearly, and it is alſo = r2/d2 × AP2/2FV, because PV = AP2/2CV nearly, and i : d = FV : CV.

*Cor.* 2. Becauſe fP : PA = Ff : FG

or FV : AV = Ff : FG nearly,

we have FG, the lateral abberration, = Ff × AV/FV, = r2/d2× AV3/2FV2, = r2/i2 × AV3/2CV2.

*Cor.* 3. Becauſe the angle F×Af is proportional to FG/FV very nearly, we have the angular abberration FAf = r2/d2 × AV3/2FV3 = r2/i2 × AV3/2CV3. In general, the longitudinal aberrations from the focus of central parallel rays are as the ſquares of the apertures directly, and as the focal diſtances inverſely ; and the lateral aberrations are as the cubes of the apertures directly, and the ſquares of the focal diſtances inverſely ; and the angular aberrations are as the cubes of the aperture directly, and the cubes of the focal diſtances inverſely.

The reader muſt have obſerved, that to ſimplify the inveſtigation, ſome ſmall errors are admitted. PV and PD are not in the exact proportion that we assumed them, nor is D*f* equal to FV. But in the ſmall apertures which ſuffice for optical inſtruments, theſe errors may be diſregarded.

This ſpherical aberration produces an indiſtinctneſs of vision, in the ſame manner as the chromatic aberration does, *viz.* by ſpreading out every mathematical point of the object into a little ſpot in its picture ; which ſpots, by mixing with each other, confuſe the whole. We muſt now deter­mine the diameter of the circle of diffuſion, as we did in the caſe of chromatic diſpersion.

Let a ray βα (fig. 5.@@) be refracted on the other side of the axis, into αHφ*,* cutting AfG in H, and draw the perpendicular EH. Call AV*a, α*Vα, Vf (or VF, or Vφ, which in this compariſon may be taken as equal) = *f, Ff — b,* and *f*E = φϰ*.*

AV2 : αV2 = Ff : Fφ (already demonſtrated) and F*φ = α2/a2b, and Ff — Fφ, (or fφ) = b — α2/a2b, = (a2b — α2b)/a2, = b/a2* × a2 *—* α2, = b/a2 × (a + α) × (a *—* α). Also Pf : PA = fE : EH, or f : a = ϰ : aϰ/f, = EH. And Pπ : Pφ = EH : Eφ, or α : f = aϰ/f : aϰ/α, = Eφ.

Therefore fφ = aϰ/α + ϰ, [(a + α)ϰ]/α, = ϰ/α × (a + α).

Therefore ϰ/α × (a + α) = b/a2 × (a + α) × (a — α), and ϰ/α = b/a2 × α*,* and ϰ/α = b/a2 *× (a — α)*, and x = b/a2 × *α(a —* α). Therefore ϰ is greateſt when α × (*a — α)* is greateſt ; that is, when α = 1/2a. Therefore EH is greateſt when Pπ is equal to the half of AP. When this is the caſe, we athave at the ſame time b/a2 × a(a *—* α) = b/a2 × 1/4a2, and ϰ = 1/4b, or EH = 1/4FG. That is, the diameter of the circle of aberration through which the whole of the refracted light muſt paſs, is 1/4 of the diameter of the circle of aberration at the focus of parallel central rays. In the chromatic aber­ration it was 1/2; ſo that in this reſpect the ſpherical aber­ration does not create ſo great confuſion as the chromatic.

We are now able to compare them, ſince we have now the meaſure of both the circles of aberration.

It has not been found poſſible to give more than four inches of aperture to an object glaſs of 100 feet focal diſtance, ſo as to preſerve ſufficient diſtinctneſs. If we com­pute the diameter of the circle EH correſponding to this aperture, we ſhall find it not much to exceed 1/120,000 of an inch. If we restrict the circle of chromatic diſperſion to 1/250 of the aperture, which is hardly the fifth part of the whole diſperſion in it, it is 1/[(62)(1/2)] of an inch, and is about 1900 times greater than the other.

The circle of ſpherical aberration of a plano convex lens, with the plane side next the diſtant object, is equal to the circle of chromatic diſperſion when the ſemi-aperture is

@@@[mu] Plate DIII.