incident rays. It is required to find the focus F of refrac­ted rays ?

Let *m* expreſs the ratio of the sine of incidence and re­fraction ; that is, let *m* be to I as the sine of incidence to the sine of refraction in the ſubſtance of the ſphere.

Then: MG : GS = sin. MSH : sin. SMG,

and: m : I = sin. SMG : sin. SMH ;

therefore: m × MG ; GS = sin. MSH : sin. SMH.

Now S, MSH : S, SMH = MH ; HS.

Therefore, finally, m*.* MG : GS = MH : HS.

Now let MS, the radius of the refracting surface, be called *a.* Let AG, the diſtance of the focus of incident rays from the ſurface, be called r. And let AH, the focal diſtance of refracted rays, be called x. Laſtly, let the sine MX of the ſemi-aperture be called *e.* Obſerve, too, that *a,* **r,** x, are to be conſidered as poſitive quantities, when AS, AG, AH, lie from the ſurface in the direction in which the light is ſuppoſed to move. If therefore the refracting ſur­face be concave, that is, having the centre on that side from which the light comes ; or if the incident rays are divergent, **or** the refracted rays are divergent ; then *a, r, x,* are nega­tive quantities.

It is plain that HS = x *—* a; GS = r — *a;* alſo AX = e2/2a nearly. HX = *a* — e2/2a. GX = r — e2/2a. Now add to HX and to GX their differences from MH and MG, which (by the Lemma) are e2/2x and e2/2r. We get MH = x — e2/2a + e2/2x, and MG = r — e2/2a + e2/2r.

In order to ſhortenour notation, make k = I/a — I/r. This will make MG = r — ke2/2.

Now ſubſtitute theſe values in the final analogy at the top of this column, viz. MH : HS = *m.* MG : GS; it becomes x — e2/2a + e2/2x : x — a = mr — mke2/2 : r — a (or ark), because k = (r — a)/ar, and ark = r — a. Now multiply the extreme and mean terms of this analogy. It is evident that it muſt give us an equation which will give us a value of *x* or AH, the quantity ſought.

But this equation is quadratic. We may avoid the ſolution by an approximation which is ſufficiently accurate, by ſubſtituting for *x* in the fraction e2/2x (which is very ſmall in all caſes of optical inſtruments), an approximate value very easily obtained, and very near the truth. This is the focal di­stance of an infinitely ſlender pencil of rays converging to **G.** This we know **by** the common optical theorem to be amr/[(m — I)(r+a)]. Let this be called φ; if we ſubſtitute k in place of I/a — I/r, this value of φ becomes = am/(m — ak). This gives us, by the by, an easily remembered expression (and beautifully ſimple) of the refracted focus of an infinitely slender pencil, correſponding to any diſtance *r* of the radiant point. For since φ *am/(m* — ak), I/φ muſt be = (m — ak)/am, = m/am — ak/am, I/a — k/m. We may even expreſs it more simply, by expanding *k,* and it becomes I/φ = I/a — I/ma — I/mr.

Now put this value of I/φ in place of the I/x in the analogy employed above. The firſt term of the analogy becomes x — e2/2a + e2/21 — ke2/2m, or x — ke2/2m. The analogy now becomes x — ke2/2m : xa = mr — mke2/2 : ark. Hence we obtain the linear equation mrx — mke2x/2 — mra + mkae2/2 = arkx — arke2/2m; from which we finally deduce

We may simplify this greatly by attending to the ele­mentary theorem in fluxions, that the fraction (x + y)/(y + y) differs from the fraction x/y by the quantity (yx — xy)/y2 ; this being the fluxion of x/y. Therefore (x + y)/(y + y) = x/y + (yx — xy)/y2. Now the preceding formula is nearly in this situation. It may be written thus; - 7 ct-—, when

*mr — ark — mke2*

the laſt terms of the numerator and denominator are very ſmall in companion with the firſt, and may be conſidered as the X and *y,* while *m r a* is the x, and *m r — ark* is the *y.* Treating it in this way, it may be stated thus :

*mra (mra'){mke2—(mr—ark)(1/2 mkae2+—*

*x = mr—a r* **r2(m—***∙ak)2*

*ark2 mra (mra^)mk-(mr—arty(mka-∖-—)*

or x = — r- + - r- — × 1/2e2

*r(m—ak) r2(m—ak)*

*mr a ma ,*

The firſt term — τ∖, or *r,* is evidently = φ*,*

*r(m—ak) m—ak*

the focal diſtance of an infinitely ſlender pencil. Therefore the aberration is expressed by the second term, which we muſt endeavour to simplify.

If we now perform the multiplications indicated by —∙ *a* r It is plain that *×mka* deſtroys the firſt term *mra× mk* of the numerator of our ſmall fraction, and there remains of this numerator *a2 r2 k* (m *a2rk2—ar2 k2* +—*~—J* 1/2 e2*,* which is equal to m2 a2*rk2 r2k2 rzh∖*

(— — T~ + ~T^)tλ

The denominator was r2 (m — *ak)2,* and the fraction *m\*a2 ∕i2 P i3 ∖ a*

now becomes ; which is evidently = T~+τ)""∙ Now recollect that

*mr myaimilz*

**i** *k3 k3 fl τ∖ P k2*

*i*  —. Therefore —=—7 ( -— — ) ≈ -r-—*~τy*

*a τ m m ∖a r ∕ m a m r*

*k2 — il k2*

Therefore, inſtead of — — , write —— — —, and we get *ma m mr*

**, r n. ∕Γ P** *P k1∖P* Z *ki mi3* **the fraction φ ~3—zr—-r-÷ — )τ= y1 ( ~~^∑3 “**

**V** *m m m r mr∕z* τ k *m m*