differs but little from that of the quantity for which it is employed. The chief change in BI is that which is deter­mined by the Lemma. Therefore take from BI the varia­tion of BH, multiplied by mBI2/BH2, which is very nearly ≡ mf2/φ. The product of this multiplication is mf2θ + mf2a/φ2. This being taken from f*,* leaves us for the value of BI

*φ*

In this value f is the focal diſtance of an infinitely ſlender pencil of rays twice refracted by a lens having no thickneſs, α(mf2/φ) is the ſhortening occaſioned by the thickneſs, and f2*(m*θ + θ') is the effect of the two aberrations ariſing from the aperture.

It will be convenient, for ſeveral collateral purpoſes, to exterminate from theſe formulae the quantities *k,* l, and φ. For this purpoſe make I/n = I/a — I/b. We have already *k =*

i L. apdl-L\_\_\_L. 1 and/ — - - -- 1

*a r ', f a and b f ~ b a 1*

1 1 χτ r 1 1 1

— ∙—~~. Now for y—- write , and we get l =

*ma nr ba n*

Therefore I/f = I/b — ml (by conſtruction, 1 I I m *mi*

page 347. Prop. II.) becomes = £ — -4.-+-, — - 4.- 1 *m—* I I

"V +7·

This laſt value of I/f (the reciprocal of the focus of a ſlen­der pencil twice refracted), viz. ffl-~∑l4-*1,* is the ſimpleſt that can be imagined, and makes *n* as a ſubſtitute for II

*I/a — I/b;* a moſt uſeful ſymbol, as we ſhall frequently find in the ſequel. It alſo gives a very ſimple expreſſion of the fo­cal diſtance of parallel rays, which we may call the princi­pal focal diſtance of the lens, and diſtinguiſh it in future by the ſymbol *p ; for* the expreſſion I/f = + I/r, becomes I/p when the incident light is parallel. And this gives us another very simple and uſeful meaſure of f; for I/f becomes = I/p + I/r. Theſe equations I/f = (m — I)/n + I/r, I/p = (m — +)/n, and I/f = I/p + I/r, deserve therefore to be made very familiar to the mind.

We may alſo take notice of another property of *n.* It is half the radius of an iſoſceles lens, which is equivalent to the lens whoſe radii are *a* and *b* .∙ for ſuppoſe the lens to be iſoſceles, that is, *a — b* ; then *n* = I/a — I/a. Now theſecond *a* is negative if the firſt be poſitive, or poſitive if the firſt be negative. Therefore I/a — I/b = (I + b)/a2 = (a + a)/a2 = 2/a, and *n = a/2.* Now the focal diſtance of this lens is (m — I)/n, and so is that of the other, and they are equiva­lent.

But, to proceed with our inveſtigation, recollect that we , , λ *m—1 f, mkt∖P ■ „ in—*i *∕ L*

had θ = —y-il·5-—— )~∙ Therefore *m* θ — — ■ ( —

*m3 ∖ r J 2 m m*

*D∖e2 λ , , m—*I z *,„ ml2∖cz*

— ~ . And θ' was = (m — I)/m *( — m3l3* + ml2/φ)e2/2. Therefore mθ + θ', the aberration (neglecting the thickneſs of the lens) is ∕ (-— ~ — w3∕j4- )-.

*, j m ∖m r i φ ∕ 2*

If we now write for k, l, and φ their values as determi­ned above, performing all the neceffary multiplications, and arrange the terms in ſuch a manner as to collect in one ſum the coefficients of *a, n,* and r, we ſhall find 4 terms for the value of *m* θ, and 10 for the value of 'θ. The 4 are deſtroyed by as many with contrary ſigns in the value of θ', and there remain *6* terms to express the value of *m*θ + θ', which we ſhall expreſs by one ſymbol 9 ; and the equation ſtands thus ;

*m—1tm3 itrlſm m-{~2 ^m2fm* 4v2+4 3w+2∖

*- m* ∖si *ani azn rnz arn* γ\*λ J *ei*

2'

The focal diſtance therefore of rays twice refracted, rec­koned from the laſt ſurface, or BI, corrected for aberration, and for the thickneſs of the lens, is f — fxx, conſiſting of three parts, viz. f, the focal diſtance of central rays; f2(mα/φ2) the correction for the thickneſs of the lens ; and f2q*,* the aberration.

The formula in the 2d par. of this col. appears very com­plex, but is of very eaſy management, requiring only the pre­paration of the ſimple numbers which form the numerators of the fractions included in the parenthesis. When the in­cident rays are parallel, the terms vaniſh which have *r* in the denominator, ſo that only the three firſt terms are uſed.

We might here point out the cases which reduce the aberration expreſſed in the formula laſt referred to, to no­thing ; but as they can ſcarcely occur in the object-glaſs of a teleſcope, we omit it for the preſent, and proceed to the combination of two or more lenſes.

*Lemma* 3. If AG be changed by a ſmall quantity Gg, BI ſuffers a change I*i,* and Gg: Ii = AG2 : BI2. For it is well known that the ſmall angles GMg and INi are equal ; and therefore their ſubtenſes G*k,* I*n* are propor­tional to MG, NI, or to AG, AI nearly, when the aperture is moderate. Therefore we have (nearly)

*Gk:* I *n* : AG : BI

I*n* : Ii = AM : BI

*Gg : Gk* = AG : AM

Therefore Gg : I *i =* AG2 : BI2

Prop. III. To determine the focal diſtance of rays refracted by two lenſes placed near to each other on a com­mon axis.

Let AM, BN (fig. 8.) be the ſurfaces of the firſt lenſe; and CO, DP be the ſurfaces of the ſecond, and let β be the thickneſs of the ſecond lens, and the interval between them. Let the radius of the anterior ſurface of the ſecond lens be *a',* and the radius of its poſterior ſurface be *b'.* Let *m'* be to I as the sine of incidence to the sine of refraction in the ſubſtance of the ſecond lens. Laſtly, let p*'* be the principal focal diſtance of the ſecond lens. Let the ex­treme or marginal ray meet the axis in L after paſſing thro' both lenſes, ſo that DL is the ultimate focal diſtance, rec­koned from the laſt ſurface.

It is plain that DL may be determined by means of a'*, b', m', p',* and CI, in the ſame manner that BI was de­termined by means of *a, b, m, p,* and AG.