The value of BI is *f — mD--fsq.* Take from this the interval δ, and we have CL = f — m... Let the ſmall part — *mα — S' — fl q* be neglected for the preſent, and let CI be ſuppoſed *— f.* As we formed φ, f, and *q,* by means of *a, b, m, n,* and r, let us now form φ', *f',*and q', for the second lens, by means of *a’, b', m', n', (* = ——)and r'. φ' will be the focal distance of a ſlender pencil refracted by the firſt ſurface, f will be the focal diſtance of this pencil after two refractions, and q' will be the coefficient of the aberration, neglecting the thickneſs and interval of the lenſes.

Proceeding in this way, DL will be = *f' — mβ(f'2/φ) — .* f'2q. But becauſe CI is really leſs than f*,* by the quantity ∞ατy + δ + f2q we muſt (by Lemma 3.) ſubtract the product of this quantity, multiplied by (which is nearly fom f' *— m f'2q'.*

By this proceſs we ſhall have

dl = f' ("j,+y1+cti) -∕∙ (,+/).

The firſt term f' of this value of DI is the focal diſtance of a ſlender pencil of central rays refracted by both lenſes, neglecting their thickneſs and diſtance ; the ſecond term, →2(⅞⅛+⅛J is the correction neceſſary for theſe circumſtances ; and the third term, — f'2 (q + q'), is the correction for the aperture 2*e.* And it is evident that *q'* is a formula precisely ſimilar to *q,* containing the ſame number of terms, and differing only by the *m', a', n',* and *r',* em­ployed in place of *m, a, n,* and r.

It is also evident, that if there be a third lens, we ſhall obtain its focal diſtance by a proceſs preciſely ſimilar to that by which we obtained DL ; and ſo on for any number of lenſes.

Thus have we obtained formulæ by which the foci of rays are determined in the moſt general terms ; and in ſuch a manner as ſhall point out the connection of the curva­tures, thickneſſes, and diſtances of the lenſes, with their ſpherical aberrations, and with the final aberration of the compound lens, and give the aberrations in ſeparate ſymbols, so that we can treat them by themſelves, and ſubject them to any conditions which may enable us to correct one of them by another.

We alſo see in general, that the correction for the thickness and diſtance of the lenſes are exhibited in terms which involve only the focal diſtances of central rays, and have very little influence on the aberrations, and ſtill leſs on the ratio of the aberrations of the different lenſes. This is a moſt convenient circumſtance ; for we may neglect them while we are determining *q* and *q',* and in determining the ratio of the focal diſtances of the ſeveral lenſes, on which the correction of the chromatic aberration chiefly depends. Therefore, in the conſtruction of a compound lens for uni­ting the different colours, we may neglect this correction for the thickneſs and diſtance till the end of the proceſs. When we apply it, we ſhall find that it chiefly affects the final focal diſtance, making it ſomewhat longer, but has hardly any influence either on the chromatic or ſpherical aberration. We do not heſitate to say, that the final for­mulæ here given are abundantly accurate, while they are vaſtly more manageable than thoſe employed by Euler or D’Alembert. We have calculated trigonometrically the progress of the rays through one of the glaſſes, which will be given as an example, giving it a very extravagant aper­ture, that the errors of the formulæ might be very remark­able. We found the real aberration exceed the aberration aſſigned by the formula by no more than 1/59th part, a diffe­rence which is quite inſignificant. The proceſs here given derives its ſimplicity from the frequent occurrence of har­monic proportions in all optical theorems. This enabled Mr Clairaut to employ the reciprocals of the radii and di­ſtances with ſo much ſimplicity and generality.

We conſider it as another advantage of Mr Clairaut’s me­thod, that it gives, by the way, formulæ for the more ordi­nary queſtions in optics, which are of wonderful simplicity,and moſt eaſily remembered. The chief problems in the elementary conſtruction of optical inſtruments relate to the focal diſtances of central rays. This determines the focal di­ſtances and arrangement of the glaſſes. All the rest may be called the refinement of optics ; teaching us how to avoid or correct the indiſtinctneſs, the colours, and the diſtortions, which are produced in the images formed by theſe simple conſtructions. We ſhall mention a few of theſe formulae which occur in our proceſs, and tend greatly to abbreviate it when managed by an experienced analyſt.

Det *m* be to I as the fine of incidence to the sine of re­fraction ; let *a* and *b* be the radii of the anterior and posterior ſurfaces of a lens ; let r be the diſtance of the radiant point, or the focus of incident central rays, and f the diſtance of the conjugate focus ; and let *p* be the principal fo­cal diſtance of the lens, or the focal diſtance of parallel rays. Make I/n equal to I/a *— I/b;* let the ſame letters a'*, b', r',* &c.

expreſs the ſame things for a ſecond lens ; and *a", b", r'',* &c. expreſs them for a third ; and ſo on. Then we have I *m—*ι l ι *m — <* i i *ni'—*1 1

*f = n* r ’ *f' n' f'' n" r,', @c.*

Therefore when the incident light is parallel, and r infinite, we have I/p = (m *— I)/n; I/p' = (m' — I)/n'; I/p'' = (m'' — I)/n'', &c.*

And when ſeveral lenſes are contiguous, ſo that their in­tervals may be neglected, and therefore I/f, belonging to the firſt lens, becomes I/r, belonging to the ſecond, we have

i i *m—*1 1 i τ

I. r'-^7, “ *η* \* r, *~ p+D*

ι I *τn— i 1 m—*i i I 1 I I

2'77≈∕, =~√^~^t *η i~D =zfi~i"p + P*

I *m''—1 nl—*I *τn—*I I I i i i

3p=~+^^Γ-÷~+P = ∕+^47 + 7^

Nothing can be more eaſily remembered than theſe formulæ, how numerous ſo ever the glasses may be.

Having thus obtained the neceſſary analyſis and formula, it now remains to apply them to the conſtruction of achro­matic lenſes ; in which it fortunately happens, that the em­ployment of ſeveral ſurfaces, in order to produce the union of the differently refrangible rays, enables us at the same time to employ them for correcting each other’s ſpherical aberration.

In the article Optics we gave a general notion of the principle on which we may proceed in our endeavours to unite the differently refrangible rays. A white or com­pounded ray is ſeparated by refraction into its component coloured rays, and they are diffuſed over a ſmall angu­lar ſpace. Thus it appears, that the glaſs uſed by Sir Iſaac