ever, be again balanced in an elliptical form, when it has ac­quired a juſt proportion oſ the axes. The proceſs for de­termining this is tedious, but preciſely ſimilar to the pre­ceding.

If the denſity of the nucleus exceed that of the fluid about

1/5½, we ſhall have r : e = G : 3P, which is nearly the form 5½ which has been determined for the earth, by the menſuration of degrees of the meridian, and by the vibration of pen­dulums. The curious reader will do well to conſult the excellent dissertations by Clairaut and Boſcovich on the Figure of the Earth, where this curious problem is treated in the moſt complete manner. Mr Bernoulli, in his dissertation on the Tides, has committed a great miſtake in this particular. On the other hand, if the nucleus be leſs denſe than the waters, or if there be a great central hollow, the elevation produced by the sun will exceed 24½ inches.

It is needless to examine this any farther. We have collected enough for explaining the chief affections of the tides.

It is known that the earth is not a ſphere, but ſwelled out at the equator by the diurnal rotation. But the change of form is ſo very ſmall in proportion to the whole bulk, that it cannot ſenſibly affect the change of form afterwards indu­ced by the ſun on the waters of the ocean. For the diſturbing force of the ſun would produce a certain protuberance on a fluid ſphere ; and this protuberance depends on the ratio of the diſturbing force to the force of gravity at the ſurface of this ſphere. If the gravity be changed in any proportion, the protuberance will change in the same proportion. Therefore if the body be a ſpheroid, the protuberance produced at any point by the sun will increaſe or diminiſh in the ſame proportion that the gra­vity at this point has been changed by the change of form. Now the change of gravity, even at the pole of the terreſtrial ſpheroid, is extremely ſmall in companion with the whole gravity. Therefore the change produced on the ſpheroid will not ſenſibly differ from that produced on the ſphere ; and the elevations of the waters above the ſurface, which they would have assumed independent of the sun’s ac­tion, will be the ſame on the ſpheroid as on the ſphere. For the ſame reaſon, the moon will change the ſurface al­ready changed by the sun, in the ſame manner as ſhe would have changed the ſurface of the undiſturbed ocean. There­fore the change produced by both theſe luminaries in any place will be the ſame when acting together as when acting ſeparately ; and it will be equal to the ſum, or the differ­ence of their ſeparate changes, according as theſe would have been in the ſame or in oppoſite directions.

Let us now conſider the moſt intereſting circumſtances of the form of an elliptical tide, which differs very little from a ſphere.

Let T (fig. 2.) be a point in the ſurface oſ the inſcribed ſphere, and let Z expreſs the angular diſtance TOQ from the longer axis of the ſurrounding ſpheroid S *m* N *q.* Let TR, TW be perpendicular to the equatorial diameter and to the axis, ſo that they are the coſine and the sine of TOQ to the radius TO or QO. Let S' *q* N' be a ſection of the circumſcribed ſphere. Draw OT cutting the ſpheroid in Z and the circumſcribed ſphere in *t.* Also let s*on* be a ſection of a ſphere which has the ſame capacity with the ſpheroid, and let it cut the radius in r. Then,

I. The elevation TZ of the point Z of the ſpheroid above the inſcribed ſphere is = Qq × cos.2Z, and the depreſſion *t*Z below the circumſcribed ſphere is = *Qq* × sine2Z. Produce RT till it meet the ſurface of the ſpheroid in V.

The minute triangle VTZ may be conſidered as a rectili­neal, right-angled at Z, and therefore ſimilar to OTR.

Therefore OT : TR = TV : TZ. But in the ellipſe OQ, or OT : TR = Qq : TV. Therefore OT2 : TR2 = Q*q :* TZ, and TZ = ¾A, = =

× cof.2 Z.

And in the very ſame manner it may be ſhown, that tZ = Qq × sin.2Z.

2. The elevation of the point T above another point T', whoſe angular diſtance TOT' from the point T is 90⁰, is Qq × cos.2Z — sin.1 Z. Call the angle QOT, Z'∙ Then Tz Z' ≈ × cof.2 Z', and TZ — T', Z', = Q~γ× cof.2 Z — cof2 Z'. But the arch QT' is the complement of QT, and therefore cos. 2Z' = sin.2 Z. Therefore TZ — T', Z' = *Qq* × cos.2 Z — sin.2 Z.

3. Qo = ⅓Qq. Forthe inſcribed ſphere is to the ſpheroid as OQ to O*q.* But the inſcribed ſphere is to the ſphere *son* as OQ3 Oo3. Therefore becauſe the ſphere *son* is equal to the ſpheroid S*q*N, we have OQ : O*q =* OQ3: Oo3, and Oo is the firſt of two mean proportionals between OQ and O*q.* But Qq is very ſmall in compan­ion with OQ. Therefore is very nearly ⅓ of Qq.

Since *son* is the ſphere of equal capacity, it is the form of the undiſturbed ocean. The beſt way therefore of con­ceiving the changes of form produced by the ſun or moon, or by both together, is to conſider the elevations or depreſſions which they produce above or below this ſurface. There­fore,

4. The elevation *r*Z of the point Z above the equicapacious ſphere is evidently = Qq × cos.2*Z —* ⅓Qq. Al­ſo the depression r' Z' of the point Z' is = Qq × ſin.2 Z' *— ⅔Qq*.

*N. B.* Either of theſe formulae will anſwer for either the elevation above, or the depreſſion below, the natural ocean : For if cos.2Z is leſs than ⅓, the elevation given by the for­mula will be negative ; that is, the point is below the natu­ral ſurface. In like manner, when ſin.2Z' is leſs than ⅔, the depreſſion is negative, and the point is above the sur­face. But if cos.2Z be = ⅓, or ſin.2Z be = ⅔, the point is in the natural ſurface. This marks the place where the ſpheroid and the equal ſphere intersect each other, viz in P', the arch P'o being 54⁰ 44' very nearly, and PS = 35⁰ 16'.

Let S repreſent the whole elevation of the pole of the ſolar tide above its equator, or the difference between high and low water produced by the ſun ; and let M repreſent the whole elevation produced by the moon. Let *x* and y repreſent the zenith diſtances of the ſun and moon with reſpect to any point whatever on the ocean. Then *x* and *y* will be the arches intercepted between that point and the ſummits of the ſolar and lunar tides. Then the elevation produced by both luminaries in that plane is S × cos.2x — ⅓S + M × coſ.2y — ⅓M ; or, more conciſely, S × cos.2 × M × cos.2y — ⅓ S + M, and the depreſſion is Sſin.2*x* + M × ſin2y — ⅔ S + M.

Let the ſun and moon be in the ſame point of the hea­vens. The ſolar and lunar tides will have the ſame axis; the coſines of *x* and *y* will each be 1, and the elevation at the compound pole will be S + M — ⅓ S + M — ⅔ S + M. The depreſſion at any point 90⁰ from this pole will be ⅓S + M, and the whole tide is S + Μ.

Let the moon be in quadrature, as in *a* (fig. 3). The appearance at *s* will be known, by conſidering that in this place the cosine of x is 1, and the coſine of *y* is o. There­fore the elevation at *s =* S — ⅓ S + M, = ⅔S — ⅓Μ. The depreſſion at *a* = S — ⅔S + M = ⅓S — ⅔M, The difference or whole tide =