For the angles BCA, BDA, ſtanding on BA, are equal. Alſo the angles IDB, μCN, are equal, being ſupplements of the angle ICB. Therefore, if BD be made radius, DA and DI are the sines of the zenith and nadir diſtances of the moon.

But BD : DA = DA : DF. Therefore DF = M × cos.2y, = the height Z*z* of the ſuperior tide. Alſo DK — M × coſ.2y', = the height *nn'* of the inferior tide.

Alſo, becauſe IA is biſected in G, KF is biſected in H, and DH = — the medium tide.

Let us trace the relation of the conſequences of the va­rious poſitions of Z and M, as we formerly conſidered the reſults of the various ſituations of the ſun and moon.

Firſt, then, let Z retain its place, and let M gradually approach it from the equator. When M is in the equator, A and I coincide with C, and the three points F, K, and H, coincide in *i.*

As M approaches to Z, A and I approach to B and D; DF increaſes, and DK diminiſhes. The ſuperior or inferior tide is greateſt when the moon is in M or in N ; and DF is then — Μ. As the moon passes to the northward of the place, the ſuperior and inferior tides both diminish till I comes to D ; at which time MQ is equal to ZP, and there is no inferior tide. This however connot happen if zP is greater than 30⁰, becauſe the moon never goes farther from the equator. M ſtill going north, we have again a perpendicular from I on BD, but below I, indicating that the inferior tide, now meaſured by DK, belongs to the hemiſpheroid next the moon. Alſo, as M advances from the equator northward, DH diminiſhes continually. Firſt, while H lies between 0 and B, becauſe G approaches 0 ; and aſterwards, when G is above 0 and H lies between 0 and D. It is otherwiſe, however, if ZQ is greater than 45⁰ ; for then DB is inclined to EQ the other way, and DH in­creaſes as the point G riſes.

In the next place, let M retain its poſition, and Z pro­ceed along the meridian.

Let us begin at the equator, or ſuppoſe Q the place of obſervation. BD then coincides with CP, and the three lines DF, DK, and DIT, all coincide with PG, denoting the two equal tides Qq and Ee and their medium, equal to either. As Z goes northward from Q, BOD detaches itſelf from COP ; the line DF increaſes, while DK and DH diminiſh. When Z has come to M, F and B coincide with A, and DK and DH are ſtill more diminiſhed. When Z passes M, all the three lines DF, DK, and DH, conti­nue to diminiſh. When Z comes to latitude 45⁰, DB is parallel to LA and EQ, and the point H coincides with O. This ſituation of Z has the peculiar property that DH (now DO) is the ſame, whatever be the declination of the moon. For IA being always parallel to DB, OK and OF will be equal, and DO will be half of DK and DF however they may vary. When Z gets ſo far north that ZPis

= MQ, the diameter bd falls on I; ſo that dk vaniſhes, and we have only *df.* And when Z goes ſtill farther north, *dk* appears on the other side of I. When Z arrives at the pole, BD again coincides with PC, D with C, and DF, DK, and DH, coincide with CG.

Theſe variations of the points F, K, and H, indicate the following phenomena.

1. The greateſt tides happen when the moon is in the zenith or nadir of the place of obſervation : for then the point B coincides with A, and DF becomes DB ; that is,

= M, indicating the full tide BB'.

2. When the moon is in the equator, the ſuperior and in­ferior tides have equal heights, = M × coſ.2 lat. For then A and I coincide with C, and the points F and K coincide in i, and D*i* is = DB × coſ.2 BDC, = M × coſ.2 lat.

3. If the place of obſervation is in the equator, the in­ferior and ſuperior tides are again equal, whatever is the moon’s declination : For then B coincides with C, and the points F, K, and H, coincide with G ; and PG — PC × coſi2 APG, = M × coſ.2 decl. moon.

4. The ſuperior tides are greater or less than the inferior tides according as the latitude and declination are of the ſame or of oppoſite names. For by making Q*ζ* = QZ, and drawing *ζ*C*n,* cutting the ſmall circle in *ß,* we ſee that the figure is reverſed. The difference between the ſuperior and inferior tides is KF, or IA × coſin. of the angle formed by IA and DB ; that is, of the angle BDδ which is the com­plement of twice ZQ ; becauſe BOC ≡ 2ZCQ. Now IA is 2GA, = 2OA × sin. 2MQ = PC × ſin. 2MQ, = M × ſin. 2 decl. Therefore the difference of the ſuperior and infe­rior tides is M × ſin. 2 declin. ſin. 2 lat.

5. If the colatitude be equal to the declination, or leſs than it, there will be no inferior tide, or no ſuperior tide, ac­cording as the latitude of the place and declination of the moon are of the ſame or oppoſite names.

For when PZ = MQ, D coincides with I, and IK va­niſhes. When PZ is leſs than MQ, the point D is between C and I, and the point Z never pasſes through the equator of the watery ſpheroid ; and the low water of its only tide is really the ſummit of the inferior tide.

6. At the pole there is no daily tide : but there are two monthly tides = M × sin.2 declin. and it is low water when the moon is in the equator.

7. The medium tide, repreſented by DH, is = Μ ×

1 + coſ. 2 lat. × coſi 2 declin.

For DH = DO + OH.

*2*

Now OH is equal to OG × coſ. GOH = OG coſi 2ZQ. And OG = OA × coſ. GOA, = OA × coſ. 2 MQ. There­fore OH = OA × coſ. 2ZQ × coſi 2 MQ. Therefore DH = OA + OA × coſ. 2ZQ × coſ. 2MQ =

Μ × ~~l~~ ~~^Ι~~~ ~~c~~~~°f~...~~~~2 co~~~~.^-ι~~~~x~~ . Let this for the future be called *m.*

*N.B.* The moon’s declination never exceeds 30⁰. There­fore coſ. 2 MQ is always a poſitive quantity, and never leſs than ½, which is the coſine of 60⁰. While the latitude is leſs than 45⁰, coſ. 2 lat. is alſo a poſitive quantity. When it is preciſely 45⁰ the coſine of its double is o; and when it is greater than 45, the coſine of its double is negative. Hence we ſee,

1. That the medium tides are equally affected by the northern and ſouthern declinations of the moon.

2. If the latitude of the place is 45⁰, the medium tide is always ½Μ. This is the reaſon why the tides along the coaſts of France and Spain are ſo little affected by the de­clination of the moon.

3. If the latitude is leſs than 45⁰, the mean tides increaſe as the moon’s declination diminiſhes. The contrary hap­pens if ZQ is greater than 45⁰. For DH increaſes or di­miniſhes while the point G ſeparates from C according as the angle COD is greater or leſs than COB ; that is, ac­cording as PCZ is greater or leſs than ZCQ.

4. When Z is in the equator, H coincides with G, and the effect of the moon’s declination on the height of the tides is the moſt ſenſible. The mean tide is then = MI + coſ. 2MQ

All that we have now ſaid may be ſaid of the ſolar tide, putting S in place of Μ.

Alſo the ſame things hold true of ſpring tides, putting M + S in place of Μ.