But in order to aſcertain the effects of declination and latitude on other tides, we muſt make a much more com­plicated conſtruction, even tho’ we ſuppoſe both luminaries in the ecliptic. For in this caſe the two depreſſed poles of the watery spheroid are not in the poles of the earth ; and therefore the ſections of the ocean, made by meridians, are by no means ellipſes.

In a neap tide, the moon is vertical at B (fig. 7. or 8.), and the ſun at ſome point of fF, 90⁰ from B. If O be this point, the conſtruction for the heights of the tides may be made by adding to both the ſuperior and inferior tides for any point D, the quantity M + S — D'F or DK × sin.22Q ſin.2dO, = M + S — tide × cos.2 MQ, as is evident.

But if the ſun be vertical at *d, d* will be the higheſt part of the circle fOF, and no correction is necessary. But in this caſe the circle of high water will be inclined to the me­ridian in an angle equal to dBO (fig. 7.), and neither the times nor elevations of high water will be properly aſcertained, and the error in time may be conſiderable in high latitudes.

The inaccuracies are not ſo great in intermediate tides, and reſpect chiefly the time of high water and the height of low water.

The exact computation is very tedious and peculiar, ſo that it is hardly poſſible to give any account of a regular progreſs of phenomena ; and all we can do is, to aſcertain the preciſe heights of detached points. For which reaſons, we muſt content ourſelves with the conſtruction already gi­ven. It is the exact geometrical expreſſion of Bernoulli’s analysis, and its conſequences now related contain all that he has inveſtigated. We may accommodate it very nearly to the real ſtate of things, by ſupposing PC equal, not to CO of fig. 4. but to MS, exhibiting the whole compound tide. And the point B, inſtead of repreſenting the moon’s place, muſt repreſent the place of high water,

Thus have we obtained a general, though not very accu­rate, view of the phenomena which muſt take place in dif­ferent latitudes and in different declinations of the ſun and moon, provided that the phyſical theory which determines the form and position of the watery ſpheroid be juſt. We have only to compute, by a very simple proceſs of ſpherical trigonometry, the place of the pole of this ſpheroid. The ſecond conſtruction, in fig. 8. shows us all the circumſtances of the time and height of high water at any point. It will be recollected, that in computing this place of the pole, the anticipation of 20 degrees, arising from the inertia of the waters, muſt be attended to.

Were we to inſtitute a compariſon of this theory with obſervation, without farther conſideration, we ſhould ſtill find it unfavourable, partly in reſpect of the heights of the tides, and more remarkably in reſpect of the time of low water. We muſt again consider the effects of the inertia of the waters, and recollect, that a regular theoretical tide dif­fers very little in its progreſs from the motion of a wave. Even along the free ocean, its motion much reſembles that of any other wave. All waves are propagated by an oſcillatory motion of the waters, preciſely similar to that of a pendulum. It is well known, that if a pendulum receive a ſmall impulſe in the time of every deſcent, its vibrations may be increaſed to infinity. Did the ſucceſſive actions of the ſun or moon juſt keep time with the natural propaga­tion of the tides, or the natural oſcillations of the waters, the tides would alſo augment to infinity : But there is an infinite odds againſt this exact adjuſtment. It is much more probable that the action of today interrupts or checks the oſcillation produced by yeſterday’s action, and that the mo­tion which we perceive in this day’s tide is what remains, and is compounded with the action of today. This being the caſe, we ſhould expect that the nature of any tide will depend much on the nature of the preceding tide. There­fore we ſhould expect that the ſuperior and inferior tides of the ſame day will be more nearly equal than the theory de­termines. The whole courſe of obſervation confirms this. In latitude 45⁰, the ſuperior and inferior tides of one day may differ in the proportion of 2½ to 1, and the tides correſponding to the greateſt and leaſt declinations of the moon may differ nearly as much. But the difference of the ſu­perior and inferior tides, as they occur in the list of Obſer­vations at Rochefort, is not the third part of this, and the changes made by the moon’s declination is not above one- half. Therefore we ſhall come much nearer the true meaſure of a ſpring tide, by taking the arithmetical mean, than by taking either the ſuperior or inferior.

We ſhould expect leſs deviation from the theory in the gradual diminution of the tides from ſpring tide to neap tide, and in the gradual changes of the medium tide by the declination of the moon ; becauſe the ſucceſſive changes are very ſmall ; and when they change in kind, that is, diminiſh after having for ſome time augmented, the change is by inſenſible degrees. This is moſt accurately confirmed by ob­ſervation. The vaſt collection made by Caſſini of the Ob­ſervations at Breſt being examined by Bernoulli, and the medium of the two tides in one day being taken for the tide of that day, he found ſuch an agreement between the progreſſion of theſe medium tides and the progreſſion of the lines MS of fig. 4. that the one ſeemed to be calculated by the other. He found no leſs agreement in the changes of the medium tides by the moon’s declination.

In like manner, the changes produced by the different diſtances of the moon from the earth, were found abundantly conformable to the theory, although not ſo exact as the other. This difference or inferiority is easily accounted for: When the moon changes in her mean diſtance, one of the neap tides is uncommonly ſmall, and therefore the ſucceſſive diminutions are very great, and one tide ſensibly affects another. The ſame circumſtance operates when ſhe changes in apogee, by reason of a very large ſpring tide. And the changes correſponding both to the sun’s diſtance from the earth and his declination agreed almoſt exactly.

All theſe things conſidered together, we have abundant reaſon to conclude, that not only the theory itſelf is juſt in principle (a thing which no intelligent naturaliſt can doubt), but also that the data which are assumed in the application are properly choſen ; that is, that the proportion of 2 to 5 is very nearly the true proportion of the mean ſolar and lunar forces. If we now compute the medium tide for any place in ſucceſſion, from ſpring tide to neap tide, and ſtill more, if we compute the ſeries of times of their occurrence, we ſhall find as great an agreement as can be defired. Not but that there are many irregularities; but theſe are evi­dently ſo anomalous, that we can aſcribe them to nothing but circumſtances which are purely local.

This general rule of computation muſt be formed in the folowing manner :

The ſpring tide, according to theory, being called A, and the neap tide B, recollect that the ſpring tide, according to the regular theory, is meaſured by M + S. Recollect alſo, that when the lunar tide only is conſidered, the ſupe­rior ſpring tide is M × sin.2, ZM (fig. 8). But when we conſider the action of two adjoining tides on each other, we find it ſafer to take the medium of the ſupe­rior and inferior tides for the meaſure ; and this is M × I + cos.2 2 ZQ × coſ. 2 MQ

*ſ* . Let this be called *m.* This