TRIGONOMETRY,

THE art of measuring the ſides and angles of triangles, either plane or ſpherical, whence it is accordingly called either Plane Trigonometry, or Spherical Tri­gonometry.

Trigonometry is an art of the greatest uſe in the mathe­matical ſciences, especially in aſtronomy, navigation, ſurveying, dialing, geography, &c. &c. By it we come to know the magnitude ot the earth, the planets and ſtars, their distances, motions, eclipſes, and almoſt all other uſeful arts and ſciences. Accordingly we find this art has been culti­vated from the earliest ages of mathematical knowledge.

Trigonometry, or the reſolution of triangles, is founded on the mutual proportions which ſubſiſt between the ſides and angles of triangles ; which proportions are known by finding the relations between the radius of a circle and cer­tain other lines drawn in and about the circle, called *cords, sines, tangents,* and seca*nts.* The ancients, Menelaus, Hippar­chus, Ptolemy, &c. performed their trigonometry by means of the cords. As to the sines, and the common theorems relating to them, they were introduced into trigonometry by the Moors or Arabians, from whom this art pasſed into Europe, with ſeveral other branches of ſcience. The Eu­ropeans have introduced, ſince the 15th century, the tan­gents and ſecants, with the theorems relating to them.

The proportion of the sines, tangents, &c. to their radius, is ſometimes expressed in common or natural numbers, which conſtitute what we call the *tables of natural sines, tangents, and ſecants.* Sometimes it is expressed in logarithms, being the logarithms of the ſaid natural sines, tangents, &c. ; and theſe conſtitute the table of *artificial sines,* &c. Laſtly, ſometimes the proportion is not expressed in numbers ; but the ſeveral sines, tangents, &c. are actually laid down upon lines of ſcales ; whence the *line of sines, of tangents,* &c.

In trigonometry, as angles are meaſured by arcs of a circle deſcribed about the angular point, ſo the whole cir­cumference of the circle is divided into a great number of parts ; as 360 degrees, and each degree into 60 minutes, and each minute into 60 ſeconds, &c. ; and then any angle is said to conſiſt of ſo many degrees, minutes, and ſeconds, as are contained in the arc that measures the angle, or that is intercepted between the legs or ſides of the angle.

Now the sine, tangent, and ſecant, &c. of every degree and minute, &c. of a quadrant, are calculated to the radius I, and ranged in tables for uſe ; as also the logarithms of the ſame ; forming the triangular canon. And theſe num­bers, ſo arranged in tables, form every ſpecies of right-angled triangles; ſo that no ſuch triangle can be propoſed, but one ſimilar to it may be there found, by comparison with which the propoſed one may be computed by analogy or propor­tion.

PLANE TRIGONOMETRY.

There are uſually three methods of reſolving triangles, or the cases of trigonometry ; viz, geometrical conſtruction, arithmetical computation, and inſtrumental operation. In the 1st method, the triangle in queſtion is conſtructed by drawing and laying down the ſeveral parts of their magni­tudes given, viz. the ſides from a ſcale of equal parts, and the angles from a ſcale of cords or other inſtrument ; then the unknown parts are meaſured by the ſame ſcales, and ſo they become known.

In the 2d method, having ſtated the terms of the propor­tion according to rule, which terms conſiſt partly of the numbers of the given ſides, and partly of the sines, &c. of angles taken from the tables, the proportion is then reſolved like all other proportions, in which a 4th term is to be found from three given terms, by multiplying the 2d and 3d together, and dividing the product by the 1ſt. Or, in work­ing with the logarithms, adding the logarithm of the 2d and 3d terms together, and from the ſum ſubtracting the logarithm of the 1st term ; then the number anſwering to the remainder is the 4th term ſought.

To work a caſe inſtrumentally, as ſuppoſe by the loga­rithm lines on one side of the two foot ſcales : Extend the compaſſes from the 1st term to the 2d or 3d, which hap­pens to be of the ſame kind with it ; then that extent will reach from the other term to the 4th. In this operation, for the ſides of triangles, is uſed the line of numbers (mark­ed. Num.) ; and for the angles, the line of sines or tangents (marked ſin. and tan.) according as the proportion reſpects sines or tangents See Sector.

In every caſe of plane triangles there muſt be three parts, one at leaſt of which muſt be a side. And then the diffe­rent circumſtances, as to the three parts that may be given,admit of three caſes or varieties only ; viz.

1st, When two of the three parts given are a side and its opppoſite angle. 2d, When there are given two ſides and their contained angle. 3d, And, thirdly, when the three ſides are given.

To each of theſe caſes there is a particular rule or proportion adapted for reſolving it by.

1st, *The Rule for the 1st Caſe,* or that in which, of the three parts that are given, an angle and its oppoſite side are two of them, is this, viz. that the ſides are proportional to the sines of their oppoſite angles ; that is,

As one side given:

To the sine of its oppoſite angle :

So is another side give:

To the sine of its oppoſite angle.

Or,

As the sine of an angle given:

To its oppoſite side;

So is the sine of another angle given:

To its oppoſite side.

So that, to find an angle, we muſt begin the proportion with a given side that is oppoſite to a given angle ; and to find a side, we muſt begin with an angle oppoſite to a given side.

*Example.* Suppoſe in the triangle BDC (fig. 1.) there be given the side BC = 106, DB = 65, and the angle BCD 31⁰ 49' given ; to find the angle BDC obtuſe and the side CD.

1. *Geometrically by Construction.*

Draw the line BC equal to 106, at C make an angle of 31⁰ 49' by drawing CD, take 65 in your compaſſes, and with one foot in B lay the other upon the line CD in D ; draw the line BD, and it is done ; for the angle D will be 120⁰ 43', the angle B 27⁰ 28', and the side DC 56.9 as was required.

**2.** *Arithmetically by Logarithms.*

As the side BD 65 - log. 1.81291

Is to fine angle C 31⁰ 49' - 9.72198

So is the side BC 106 - 2.02531

11.74729 1.81291

To sine angle D 120⁰ 43' - 9.93438