|  |  |
| --- | --- |
| **To find DC.** | **180.0** |
| As sine ang. C 31⁰ 49' : 9.72198 | The supp. 59. 17 of ang. D. |
| Is to the side BD 65 : 1.81291 |  |
| So is sine ang. B 27.28 : 9.66392 | 120.43 angle D. |
|  | 31.49 angle C. |
| 11.47683 |  |
| 9.72198 | 152.32 their ſum. |
| To the side DC 56.88 175485 | 180.0 |
|  | 152.32 ſum ſubt. |
|  | 27.28 angle B. |

Here it may be proper to obſerve, that if the given angle be obtuſe, the angle ſought will be acute ; but when the given angle is acute, and oppoſite to a lesser given side, then the required angle is doubtful, whether acute or obtuſe ; it ought therefore to be determined before the operation. For it is plain the above proportion produces 59⁰ 17' for the re­quired angle ; but as it is obtuſe, its ſupplement to 180 de­grees muſt be taken, viz. 120⁰ 43'.

*By Gunter.*

“ The extent from 65 to 106 on the line of numbers will reach from 31⁰ 49' to 59⁰ 17' on the line of sines.”

2dly, “ The extent from 31⁰ 49' to 27⁰ 28' on the line of sines will reach from 65 to 56.88 on the line of numbers.”

Case II. When there are given two ſides and their con­tained angle, to find the rest, the rule is this :

As the ſum of the two given ſides :

Is to the difference of the ſides : :

So is the tangent of half the ſum of the two oppoſite angles or cotangent of half the given angle :

To tang. of half the diff. of thoſe angles.

Then the half diff. added to the half ſum, gives the great­er of the two unknown angles ; and ſubtracted leaves the leſs of the two angles.

Hence, the angles being now all known, the remaining 3d side will be found by the former caſe.

*Example.* The side BC = 109, BD = 76 (fig. 2.), and the angle CBD 101⁰ 30' given, to find the angle BDC or BCD, and the side CD.

**I.** *Geometrically by Construction.*

Draw the line BC 109, and BD, ſo as to make an angle with BC of 101⁰ 30', and make BD equal to 76 ; join BC and BD with a right line, and it is done ; for the angle D being meaſured by the cord of 60 , will be 47⁰ 32', angle C 20⁰58', and the side DC 144.8, as was required.

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| 2. *Arithmetically by Logarithms.* | | | | | | |
| Side BC 109 | 109 | 180⁰ | 0' | | | |
| BD 76 | 76 | 101 | 3⁰ | | | |
| Their ſum 185 | 33 | their diff. 78 | 30 ſum of the ang. D and C. | | | |
|
|  | | 1/2 Sum 39 | 15 then | |  |  |
| To find the angles D and C. | | |  | |  |  |
| As the ſum of the ſides BC and BD = | | | 185 |  | 2.26717 |  |
| Is to their difference | | | 33 |  | 1.51851 |  |
| So is tang. of 1/2 the ſum of the angles C and D 39⁰ 15' | | | | | 9.91224 |  |
|  | | | | | 11.43075 |  |
|  | | | | | 2.26717 |  |
| To the tang of 1/2 the diff. of the angles C and D 8⁰ 17' | | | | | 9.16358 |  |
| To half the ſum of the angles D and C | | | |  | 39⁰ | 15 |
| Add half the difference of the angles C and D | | | |  | 8 | 17 |
| Gives the greater angle D | | | | | 47 | 32 |
| Subtracted, gives the lesser angle C | | | | | 30 | 58 |

|  |  |
| --- | --- |
| **To find DC.** |  |
| **As fine angle D 47⁰ 32'** | **9.86786** |
| Is to the side BC 109 | 2.03743 |
| So is sine angle B 101⁰ 30' | 9.99119 |
|  | 12.02862 |
|  | 9.86786 |
| To the side DC required 144.8 | 2.16076 |

3. *By Gunter.*

1ſt,“ The extent from 185 to 33 on the line of numbers will reach from 39⁰ 15' to 8⁰ 17' on the line of tangents 2dly, The extent from angle D 47⁰ 32' to 78⁰ 30' (the ſupple­ment of angle B) on the line of sines, will reach from the side BC 109 to 144.8, the side DC required, on the line of numbers.”

Case III. Is when the three ſides are given, to find the three angles ; and the method of reſolving this caſe is, to let a perpendicular fall from the greateſt angle upon the oppoſite side or baſe, dividing it into two ſegments, and the whole triangle into two ſmaller right-angled triangles : then it will be,

As the baſe or ſum of the two ſegments :

Is to the ſum of the other two ſides :

So is the difference of thoſe ſides:

To the difference of the ſegments of the baſe.

Then half this difference of the two ſegments added to the half ſum, or half the baſe, gives the greater ſegment, and ſubtracted gives the leſs. Hence, in each of the two right-angled triangles, there are given the hypothenuſe, and the baſe, beſides the right angle, to find the other angles by the firſt caſe.

*Example.* The ſides BC (fig. 3.) = 105, BD = 85, and CD = 50, given to find the angles BDC, BCD, or CBD.

**1.** *Geometrically by Construction.*

Draw the line BC equal to 105, take CD 50 in your compaſſes, and with one foot in C deſcribe an arch ; then take BD 85 in your compaſſes, and with one foot in B cut the former arch in D, join BD and DC, and it is done ; for the angle B, being meaſured, will be found 28⁰ 4', angle C 53⁰ 7', which being added together, is 81⁰ 11', their ſum ſubtracted from 180, leaves angle D 98⁰ 49' as was required.

**2** *Arithmetically by Logarithms.*

The two ſhorteſt ſides are BD ( — 85) and CD (=50), the ſum of which is 135, and their difference 35. The ſegments of the baſe BC are found in this manner :

As the side BC = 105 log. 2.02119

Is to the ſum of the ſides BD & DC = *135,* 2.13033

So is their difference = 35, 1.54407

To the difference of the ſeg. of BC = 45, 1.65321

Thus the ſum and difference of the ſegments of the baſe BC being known, we have only to add half this ſum = 521/2 to half the difference = 221/2, and we ſhall obtain the greater ſegment, which is = 75 ; which ſubtracted from 105, gives 30 = the ſmaller ſegment. Then

To find the angle BDA.

As the hypothenuſe BD = 85 log. 1.92942

Is to radius = 10.00000

So is the greater ſegment = 75, 1.87506

To the ſum of the angle BDA = *9,94564*

The angle BDA therefore is equal to 61⁰ 56'

Let us now find the angle ADC, which is done thus. As the hypothenuſe DC = *50* log. 1.69897

Is to radius = 10.00000

So is the ſmaller ſegment = 30, 1.47712

To the sine of ADC = 9.77815

The angle ADC therefore is equal to 36⁰ 53', and the whole angle BDC = 98⁰ 49'.