it will be, As radius is to the co-ſine of the hypothenuſe, ſo is the tangent of either angle to the co-tangent of the other angle.

As the ſum of the sines oſ two unequal arches is to their difference, ſo is the tangent of half the ſum of thoſe arches to the tangent of half their difference : and as the ſum of the co-ſines is to their difference, ſo is the co-tangent of half the ſum of the arches to the tangent of half the dif­ference of the ſame arches.

Theorem V. In any ſpherical triangle ABC (fig. 9 and 10.) it will be, As the co-tangent of half the ſum of half their difference, ſo is the co-tangent of half the baſe to the tangent of the diſtance (DE) of the perpendicular from the middle of the base.

Since the last proportion, by permutation, becomes co- tang.(AC + BC)/2 : co-tang. AE : : tang. (AC ­­­— BC)/2 tang.

DE, and as the tangents of any two-arches are, inversely, as their co-tangents ; it follows, therefore, that tang. AE : tang. (AC + BC)/2 : : tang. (AC — BC)/2 : tang. DE; or, that

the tangent of half the baſe is to the tangent of half the ſum of the sides, as the tangent of half the difference of the sides to the tangent of the diſtance of the perpendicular from the middle of the baſe.

Theorem VI. In any ſpherical triangle ABC (fig. 9.) it will be, As the co-tangent of half the ſum of the angles at the baſe is to the tangent of half their difference, ſo is the tangent of half the vertical angle to the tangent of the angle which the perpendicular CD makes with the line CF biſecting the vertical angle.

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| The Solution of the Cases of right-angled ſpherical Triangles, (fig. 8.), | | | |
| Caſe | Given | Sought | Solution |
| ***I*** | The hyp. AC and one angle A | The oppoſite leg BC | As radius : sine hyp. AC : : sine A : sine  BC (by the former part of theor. 1.) |
| ***2*** | The hyp. AC and one angle A | The adjacent leg  AB | As radius : co-ſine of A : : tang. AC. : tang. AB (by the latter part of theor. 1.) |
| *3* | The hyp. AC and one angle A | The other angle C | As radius : co-ſine of AC : : tang. A : co­tang. C (by theorem 4.) |
| 4 | The hyp. AC and one leg AB | The other leg  BC | As co-ſine AB : radius : : co-ſine AC : co-ſine BC (by theorem 2.) |
| 5 | The hyp. AC and one leg AB | The oppoſite an­gle G | As fine AC : radius : : sine AB : sine C (by the former part of theorem 1.) |
| *6* | The hyp. AC and one leg AB | The adjacent an­gle A | As tang. AC : tang. AB : ; radius : co- ſine A (by theorem 1.) |
| 7 | One leg AB and the adjacent angle A | The other leg BC | As radius : sine AB : : tangent A : tan­gent BC (by theorem 4.) |
| 8 | One leg AB and the adjacent angle A | The oppoſite an­gle C | As radius : sine A : ; co-ſine of AB : co- ſine of C (by theorem 3.) |
| 9 | One leg AB and the adjacent angle A | The hyp.  AC | As co-ſine of A : radius : : tang. AB : tang. AC (by theorem 1.) |
| **10** | One leg BC and the oppoſite angle A | The other leg  AB | As tang. A : tang. BC : : radius : sine AB (by theorem 4.) |
| **I I** | One leg BC and the oppoſite angle A | The adjacent an­gle C | As co fine BC : radius : : co-ſine of A ; line C (by theorem 3.) |
| **12** | One leg BC and the oppoſite angle A | The hyp. AC | As sine A : sine BC : : radius : sine AC (by theorem 1.) |
| 13 | Both legs  AB and BC | The hyp.  AC | As radius : co-ſine AB : : co-ſine BC : co- ſine AC (by theorem 2.) |
| 14 | Both legs  AB and BC | An angle, ſuppoſe A | As sine AB : radius : : tang. BC : tang. A (by theorem 4.) |
| 15 | Both angles A and C | A leg, ſuppoſe AB | As sine A : co-ſine C : : radius : co-ſine AB (by theorem 3.) |
| 16 | Both angles  A and C | The hyp. AC | As tang. A : co-tang. C : : randius ; co- ſine AC (by theorem 4. ) |

*Note,* The 10th, 11th, and 12th cases are ambiguous ; ſince it cannot be determined by the data, whether A, B, C, and AC, be greater or leſs than 90 degrees each.