1 ■ 2 ι 2 ∙ 3 3 ∙ 4 n(a + 1)

Γτ2 + **Tτ2 ÷** Γ72 ” ' + + i ∙ 2 a (a + 1) (a + 2)

18 1 ∙ 2 ∙ 3

The next series, which has -for its gene­

ral term, as well as all that succeed, will be found to have the very same property, as may be proved as follows. Let *p* denote any term of the series of natural numbers 1, 2, 3, &c. Then, because

, + p \*>—1

*p* + l *p* + 1

if we multiply these equals by the product of all the factors

r, —E T\_, 4c. to , we get

»(» + !)(» + 2)∙∙ ∙ (p+p— 1)

1 ∙ 2 ■ 3 ∙ ∙ ∙ jd

( p(p+ l)(f+ 2)∙∙∙(p + P)

I 1 ∙ 2 ∙ 3 ∙ ∙ Q> + 1)

**J** (»-1)»(»+1)···(» + ρ-1)

**I** l∙2∙3∙∙∙(p+ 1)

Now, if in this identical equation we substitute the num­bers 1, 2, 3, &c. to *n* successively, for », the results obtain­ed from its first member

ρ(»+1)(» + 2)···(»+ρ—1)

1∙2∙3 ∙ ∙*∙p ’*

will be a series having this function for its general term, and the terms of which will evidently be the difference be­tween the terms of another series having the first part of the second member of the equation, viz.

v (v + 1) (v + 2) ∙ ∙ ’ *(v +p)* 1∙2∙3∙∙∙Q>+1) ’

for its general term : Hence it will happen, as in the two foregoing problems, that the sum of all the terms of the former series will be equal to the last term of the latter ; which conclusion may be expressed in the form of a theo­rem, as follows :

THeorem. *The sum of* n *terms of a series having for its general term the function*

»(»+1)(» + 2)···(»+ρ-1) l∙2∙S∙∙∙p

*is equal to*

a(a + l)(a + 2)∙∙∙∙(a + *p)* l∙2∙3∙∙(7+tr ∙

Or, setting aside the denominators of the terms, we may express the theorem thus : *The sum of* n *terms of a series, having far its general term the expression*

»(»+ 1)(» + 2)···(»+ρ\_1),

*is equal to*

w(a + O(» + 2) ···(»+/>)

*P + 1*

We shall give a few particular cases of this formula.

I. l+2 + 3 + 4∙∙∙ + \* = "(w+υ∙

II. l∙2 + 2∙3 + 3∙4 + 4∙5∙∙∙ + n(>I+ 1)

\_a(a+l)(a + 2)

- 3

III. l∙2∙3 + 2∙3∙4 + 3 ∙ 4 ∙ 5 ∙ ∙ ∙+a(n+1) (a + 2)

n(n+l)(n+2)(n+3)

- 4

(10.) By means of the above general theorem, we may find the sum of any number of terms of a series composed

of the powers of tiιe terms of an arithmetical progression, the general term of which will, in the simplest case, be **ti,** *p* being a given number. The manner of doing this will appear from the following problems.

■Prob. III. It is proposed to find the sum of a terms of the series of squares 1 + 4 + 9 + 16 + 25 +, &c. or 1’ + 2ii +3s + 4\* + 5! +, &c.

The general term of this series being r\*, we put it un­der this form, *v (v* + 1) — *v ;* hence we get, by substitut­ing 1, 2, 3, &c. for *V,*

1\*= l∙2-I,

2» = 2 ∙ 3 — 2,

33 = 3 ∙ 4 — 3,

4! = 4 ∙ 5 — 4,

n2 = n(a + l)- *η.*

Therefore, adding, we find I« + 2» + 3» + 4’ ∙ ∙ ∙ ∙ -t- n\*

il∙2 + 2∙S.+ S∙4 + 4∙5∙∙∙+ a(a + l) t — (l + 24-8-f-4,∙,4∙n).

But by the general theorem (9.)

l∙2+2∙3 + 3∙4∙∙∙ + a(a+l) = -~~C” + Ό <~~~~w~~~~+~~J),

and 1 + 2 + 3 + 4 ∙ ∙ ∙ + *n = —) .*

therefore l2 + 22 + 32 + 42 ∙ ∙ ∙ + *ηi* \_ ” (" + l)(n + 2) a(n + l)

3 2

\_ a (a + 1) (2a + 1)

6

We might have arrived at the same conclusion by consi­dering, that since »‘, the general term of the series, is equi­valent to *v (v* + 1 *) — v,* the series must be the difference between two others, one having *υ* (*υ* + 1) and the other *υ* for its general term ; for the sake of perspicuity, however, we have put down the terms of all the three series.

Prob. IV. It is proposed to find the sum of *n* terms of the series

l3 + 2’ -∣- S3 + 43 + 5’ +, &c.

The general term in this case is «’ ; now, to transform this function, so as to deduce the sum of the series from the general theorem, we assume

*υ*3 = *v* (» + 1) *(v* -j- 2) + Α» (» + 1) + Be, where A and B denote quantities which are to have such values as shall render the two sides of the equation identi­cal, whatever be the value of *υ*. Taking now the product of the factors, we have

*v3 = v3* + (A + 3) »' + (A + B + 2) *v,* therefore, by the theory of indeterminate co-efficients (Al­gebra, § 159),

A + S = 0, A + B+ 2 = 0t hence we find A = — 3, B = — A — 2 = 1; thus it ap­pears that, *v* being any number whatever,

*υ3 = v (υ* -J- 1) (» + 2) — 3» (» + I) + ».

Now, let S denote the sum of *n* terms of the series under consideration, which has *υ3* for its general term, and put P, Q, R for the like sums of the three series whose gene­ral terms are the functions *υ* (*υ* + 1) (*υ* + 2), *υ*(*υ* + 1), and » respectively ; then it is evident that S = P — 8Q + R. But by the theorem (9.)

\_ a (a + 1) (a + 2) (a + 3),

4

n \_ a (a + 1) (a + 2)

3 —,

\_a(a + l)

**κ- 2 ,**