Taking now the fluxions, and substituting instead of *dp* the expression it was put to represent, we get

*xn ~ ,ds* + (» — 1 ) *sxm ~ tdx* = (n-l)\*-¾te + Ί-—-+(Τ=-

and this, after reduction, becomes

*ds+--sdx = —- +T\_\_+\_\_.*

This fluxional equation being of the first degree and first order, its primitive equation may be found (from the gene­ral formula given in Fluxions, § 171) to be

1 V *cf* z ι∖ „ sj *,mx"~'dx xndx* 1

i= F=7∙**×√i("“υ**~ dx+-∣⊂7~ + **J5**

and this again, by remarking that *p(n — i)xπ~ tdx = x^ ~ \*,* and that

∕>m.r" ^^ *ldx mxn ∕∙ mx^dx*

*J ∖-x* **-n(l —a·)-.∕∏(l-.r)∙,**

may be reduced to

— ι **\_l mx** *∣ w — m Γ χndχ*

« (I — *x) mn ~lJ* (1 — je)s'

***/» .}'il(lfC***

The remaining fluent ∕ — -3 may be found by § 101,

Flux∣0N3, and it must be so taken, that after being multi­plied by - it shall vanish when *x = Ö ;* for then this

hypothesis will make the whole function which expresses the value of *s* vanish, except its first term 1, as it ought to do.

*Example.* Let us suppose *n* = 2, then

*j» x dx t x* ,a, .. ,

∕(TΞΞ7y = \* + Γ=ι + 21°β∙<1-a\* and

2 — *m ∕∙ xidx* (2—*m)x*

*2x J* (T∑∑^ij5 - 8(1—»)

+ 10g∙ (1 - \*),

the fluent being here taken as directed. In this case, after collecting the terms, we get or

*m* m+l m+2

1 + —— » + —,\*s+ —ÿ- ·»’ +, &c.

log. (!-»).

(24.) There is a branch of the doctrine of series which is of considerable importance in pure mathematics, as well as in many physical inquiries, and in the science of astronomy ; it is called the *Interpolation* of series.

To interpolate a series is to interpose among its terms others which shall be subject to the same law, or which shall be formed in the same manner as the original terms of the series ; or, in other words, it is to find the value of one or more terms by means of others which are given, and which may be either at equal or unequal intervals from one another, the places of the given terms as well as of those sought be­ing supposed known.

It is easy to see that this problem may be applied to the construction of logarithmic tables ; for we may regard the logarithms of the natural numbers 1, 2, 3, 4, &c. *ad infini- tutn,* as the terms of a particular series of which the num­bers themselves are the indices ; thus having given the lo­garithms of some numbers, we may by interpolating deduce **from** them the logarithms of others.

Again, in astronomy we may consider the numbers which express the successive observed ∣κ>sitions of a celestial body

as the term3 of a series, their indices being the intervals of time between the observations and some assumed epoch, and the problem we are considering will enable us to de­termine the position at any instant different from the times of actual observation, provided the intervals between the observations be small, and the instant for which the posi­tion is sought not very remote from those at which the ob­servations were made.

(25.) To illustrate the nature of the problem to be re­solved, let us consider some particular case, as, for example, the arithmetical series

*a, a + d, a + 2d, a + 3d, a -⅛- id,* &c.

Let *t* and *t* be two givcn terms of the series which are at any distance from one another, and let *n* and *n,* be their indices, or numbers which denote their places in the series. Also let *y* be any term whatever, and *x* its index. Then, by the nature of an arithmetical series,

*r = a + (n—* **l)rf, <z = α + (n'— l)rf, y= a + (at— l)d.**

Now, as there are here three equations, each involving the quantities *a* and *d,* we may eliminate both these quantities by the common rules (Algebra, sect, vii.) ; and this being done, we get

*(x—ri)* (ι'—*t)* = (»' ->- n) (y — <,) ; and hence we find this expression,

*y=τ∑∑\*l+\*≡nt'*

which is a general formula for interpolating any arithmeti­cal series ; and it is observable, that it is entirely independ­ent both of the first term and common difference.

*Example.* The 7th term of an arithmetical series Ls 15, and the 12th term is 25 : it is required to find the 10th term.

Here *n* = 7, *n'* = 12, *x* = 10 ;

*t =* 15, <' = 25, y is sought.

Therefore, by the formula,

2 3

w = -χl5 + -×25=2l, the **answer.**

5 o

(26.) The mode of investigation by which we have found a formula for the interpolation of an arithmetical series will apply also to others, if the law according to which the terms arc formed be known ; in general, however, the law of a series to be interpolated is either not known, or it is not taken into account, and we only consider the absolute mag­nitudes of certain terms, and the numbers expressing their places in the series. To resolve the problem generally with these *data,* it is usual to proceed as follows: Let a straight line AB, and a point A in it, be assumed as given in posi­tion, and let there be taken the segments AD, AD', AD”, AD’", Ac., proportional to the numbers denoting the places of the terms of a series reckoned from any term assumed as a fixed origin ; and at the points D, D', D,, let there be erected perpendiculars proportional to the terms themselves. Let us now suppose a curve to pass through C, C', C”, C'", &c. ; then, if it be so chosen that its curvature may vary gradually in its progress from point to point, without any abrupt changes of inflection, and, moreover, if the terms (which we may suppose to be either at equal or unequal dis­tances) are pretty near to one another, it is easy to con­ceive, that if AP be taken equal to the number expressing