DF and CE to intersect each other in B ; then on DB pro­duced take BV = DF, and on be take BI = EC ; com­plete the parallelogram VBIH, and BH will represent, in quantity and direction, the resultant of the whole of the direct and vertical resistances against the fore and after parts of the ship ; and BH, multiplied into GM, GM being drawn from the centre of gravity G of the ship, perpendi­cular to BH produced, will represent its moment to make the ship revolve round its centre of gravity. The centre of effort of the sails must therefore be at such a height that the moment of the wind, estimated from the centre of gra­vity of the ship, may be equal to this moment. If HM re­present the line of direction of the effort of the wind on the sails, the force of the wind acting in that direction will be represented in quantity by HB ; but as the action of the wind is horizontal, and is equal to the horizontal effort of the water, if BH be resolved into BN and NH, then BN represents, in quantity and direction, the horizontal resist­ance of the water, and NB in the same manner represents the horizontal effort of the wind ; and if GO be drawn from G perpendicular to the horizon, meeting HB in O, we have, from similar triangles, HBN and OGM, NB × GO = HB × GM ; that is, O, the point in which a vertical line pass­ing through the centre of gravity of the ship intersects the direction of the resultant of the resistances of the water against the fore and after parts of the ship, is the correct height at which the centre of effort of the sails should be placed, that the ship’s horizontal water-line, when she has acquired an uniform velocity, may not be affected by any change in the force of the wind.

This point O does not fulfil the conditions of Bouguer’s *point velique,* as it only determines the position of the centre of effort of the sail as to height above the centre of gravity of the ship ; for the moment of sail acting in a horizontal direction, estimated above that centre of gravity, will be the same at whatever point in a horizontal line passing through O the centre of effort may be placed ; therefore this point O may be more properly called the height of sail. The position of the centre of effort of the sails in a horizontal line at this height of sail, will depend on the considerations which have been explained in a former part of this article. It may therefore differ very considerably from that deter­mined by Bouguer, not only in its vertical, but in its hori­zontal position.

From this investigation of Chapman’s, it evidently ap­pears, that unless BH coincides with NB, that is, unless the resultant of the resistances of the water is horizontal, there will be a force NH or HN acting in a vertical di­rection, either upwards or downwards, at the centre of gra­vity of the ship, according as the positive or negative ver­tical resistances are the greater ; for this force, acting in a vertical direction, cannot be derived from the direct resist­ances, which act horizontally ; and since the whole force of the wind acts in a horizontal direction, and is destroyed by the horizontal effort of the water, no part of its force can be employed in a vertical direction affecting the action of HN or NH. This force will therefore act to increase or di­minish the displacement of the ship when in motion, ac­cording as the negative or positive vertical resistances are the greater, that is, the quantity by which the displacement would be increased by the diminution of the vertical pres­sure upwards, incidental to the motion of the vessel, will be diminished by the action of the force NH ; but unless NH is greater than the diminution of the vertical pressure upwards, it will have no effect on the position of the centre of gravity of the displacement, and therefore none on the longitudinal inclination of the ship. The force HN, acting in conjunc­tion with the diminution of the vertical pressure upwards, will affect the position of the centre of gravity of the dis­placement in the same manner as that affects it, that is, de­pendent on the relative form of the body above the water.

Therefore in this case the ship may have a slight tendency to longitudinal oscillation, even though the centre of effort of the sails be placed at the height of sail, as determined by Chapman ; but this will not affect the correctness of Chapman’s principle, and a ship may be easily constructed with such a form at the parts about the surface of the wa­ter, that this inconvenience will not occur.

Now, if we suppose BV and BI to be given in position, the force NH or HN will depend, in quantity and direc­tion, on the proportion between BV and BI, that is, on the proportion of DF to EC. Now when BH coincides with NB, or when the direction BH of the resultant of the water is horizontal, the force NH or HN vanishes ; there­fore, if we suppose HB to coincide with BN, then HB is parallel to CD; and the angle VBH will be equal to the angle BDC, and the angle BCD will equal the angle HBC = VHB. Now VH:VB::sin. VBH:sin. VHB::sin. BDC: sin. BCD ; and since VH = BI = EC, and B V = DF, there­fore EC : DF:: sin. BDC: sin. BCD; consequently we have, that the positive and negative vertical resistances are equal to one another, and the direction of the resultant of the re­sistances of the water will be horizontal, when the result­ant of the direct and vertical resistances of the water on the bows of the vessel is to the resultant of the direct and vertical resistances on the stern, inversely as the sines of the angles which the respective directions of these resul­tants make with a horizontal line.

The extremities of a vessel of the usual form may, for the purpose of determining the proportion between the di­rect and vertical resistances which they experience, be con­sidered as planes moving obliquely in a fluid, and conse­quently the proportions between the direct and vertical re­sistances will depend on the angles of inclination which the surfaces of these extremities make with the direction of the vessel’s motion, that is, with a horizontal line ; and the sum of the direct resistances on either extremity will be to the sum of the vertical as the cosine to the sine of the angle of inclination ; consequently, as long as the inclinations of the bow and stern to a horizontal line remain unchanged, this proportion between the direct and vertical resistances ex­perienced by those parts respectively, that is, the propor­tion of DK to KF, and of LC to EL, will be invariable; and therefore, as far as these considerations are involved, the directions of the resultants DF and EC will remain constant, whatever alteration may take place in their re­lative proportion to each other, arising from any increase or diminution in the velocity of the vessel.

Since, when the direction of the resultant of the water is horizontal, EC:DF:: sin. BDC: sin. BCD, then EC- sin. BCD = DF ∙ sin. BDC ; now let us suppose the proportion of DF to EC to be altered, so that EC·sin. BCD will be greater than DF·sin. BDC, that is, let us suppose the comparative proportion of DF to EC to be in­creased.

Produce DV to P, and make BP to BI in the increased proportion of DF to EC, complete the parallelogram BPQI, and draw the diagonal BQ ; BQ will represent the direction of the resultant of the resistance of the water, after the al­teration of the proportion between DF and EC.

Produce QB to S, then the angle PBQ = the angle DBS, and since BP is parallel to IQ, the angle PBQ is equal to the angle IQB, and the angle IHB is greater than the angle IQB ; consequently the angle DBM, which is equal to the angle IHB, is greater than the angle IQB, that is, the angle DBM is greater than the angle DBS, and S, the point in which the direction QB cuts the vertical line GO, will be within or below the point O ; therefore GS will be less than GO. If EC, that is, BI, had been in­creased in proportion to DF or BV, the point S would, in the same manner, have been found to be above the point O ; consequently, from this we may deduce the following