The position of the fulcrum of the lever being now known, we can proceed to ascertain the effects of the various forces.

Returning, for the sake of simplicity, to the rectangular beam ; the sum of the repulsions *i*I will be represented by the triangle G*f*F : but the rectangle under GF and *Ff* would measure the entire strength of the under half of the beam, and therefore the force actually exerted when the beam is about to be broken across, is just half of the abso­lute strength of the beam ; one quarter being exhibited as attraction, another quarter as repulsion. These forces act at different distances from the fulcrum G, and it is well known that the influence of a number of weights in turning a lever round is the same as if all these weights were to act at their common centre of gravity ; so that to find the en­tire action in this case, we have to suppose one fourth of the whole strength of the beam to act at the distance 2/3 of GF or 1/3 of EF from the fulcrum G, and another quarter of the strength at 2/3 of GE. Now, if *s* be the strength of one square inch of the beam, and if D, B, and L be its depth, breadth, and length, measured in inches, DBs will be the absolute strength of the whole beam ; and therefore the tendencies of the above forces to straighten the beam will be 1/4DB*s*×1/3D +1/4DB*s*×1/5D;

that is, 1/6D2 ∙ B ∙ *s.* And again, the tendency of the weight W to bend the beam is W L ; so that

1/6 D2∙B∙s= WL,

or 6L : D : : DBs : W;

that is, *as six times the length of the beam is to its depth, so is the absolute strength of the beam to the weight which it can carry at the free end.*

Throughout this investigation we have supposed that the force needed to extend or compress a fibre is exactly pro­portional to the quantity of extension or compression. This hypothesis, though not perhaps strictly true, and though it certainly errs when we approach to dislocation or fracture, is yet confirmed by all experiments when the extensions have been kept within the limits of safety : the results of this hypothesis therefore are what must guide us in forming any structure.

It may now be interesting to inquire into the strength of a beam when bent in different directions.

EF being, as before, the cross section of an irregular beam, let that beam he bent in the direction OG, the axis of flexure being OH perpendicular to OG ; and let us con­sider the action of a small portion *ds* of the surface situated at I. Having drawn the perpendiculars IK and IL, it is clear that the compression of the fibre I must be propor­tional to KI ; and therefore the force exerted by it may be denoted by C ∙ IK ∙ *ds,* C being some constant depending on the nature of the material and the degree of flexure. This force, conceived to act at the extremity of the lever KI, will tend to bend the beam round the axis OH ; but again acting at the end of the lever LI, it will tend to bend the beam on OG as an axis. Putting 2 to denote the integral for the entire surface, C∙Σ·IK2*ds* will be the entire tendency to rectify the form of the beam, and CΣIK ∙ IL∙*ds* will be the entire tendency to take a flex­ure in a plane at right angles to that which it actually has. A beam therefore will not bend in the direction in which the pressure is applied to it unless 2∙IK∙ IL*ds* = 0. This is a circumstance overlooked in all treatises on flex­ure; but it is one that must be carefully attended to in practice. It may easily be illustrated thus : Take a thin slip of wood, such perhaps as is used for Venetian blinds, and fix it in a vice so that while its lcngth is horizontal its flat sides may be inclined at a considerable angle. Attach now a weight to the free end, and it will be found that that end does not descend vertically, but that it moves obliquely, the flexure not happening in that direction in which the force is applied. The force necessary to bend the beam in the plane of OG is not a force in that

direction, but is the resultant of two forces, Σ IK2 ∙ *ds* in the direction OG, and 2 ∙ IK ∙ IL ∙ *ds* in the direction OH. For the present we shall call ΣIK2*ds* the *stiffness in the direction* OG, and ∑∙ IL2*ds* of course the stiffness in the direction OH. The sum of these two stiffnesses is mani­festly Σ · OI2*ds*, which is a constant quantity, depending not at all upon the directions of OG, OH, but only on the form of the cross section : hence follows the remarkable law, that the sum of the stiffnesses of a beam in two directions perpendi­cular to each other is constant ; and that therefore, what­ever may be the form of the beam, its directions of greatest and least stiffness are always perpendicular to each other.

For the purpose of discovering in what directions the greatest and least stiffness lie, let us refer all the points in the cross section to the axes OX and O Y, putting the angle XOG = *φ.* We have then IK = *x* cos. *φ -∣- y* sin. *φ,* IL =— a: sin. *φ.* p *y* cos. p ; and thus the tendency of the beam to redress itself in the direction GO is 2∙(*x* cos. P + *y* sin. *φ ds,* while the deflecting tendency in the direction HO is Σ· (\* cos. *φ* -f- *y* sin. *φ) (— x* sin. *φ* + *y* cos. *φ) ds.* Regarding *φ* as the variable quantity, and dif­ferentiating the former for the purpose of discovering its maximum, we obtain 2∙(z cos. 0 + y sin. *φ) (—x* sin. φ*+ y* cos. *φ)* ds= 0; now it will be observed that this expression is just that for the deflecting tendency, and hence this law,

*That when a beam is bent in the direction of greatest or of least stiffness, the pressure to be applied is exactly in the direction of the betiding.*

The value of *φ* may easily be found from the above equation ; the result is

*„ 2 S∙ x∣∕ ds*

tan. 2 ® = ——∙—*'■—-χ-r-. r Sx‘ds — Syds*

The lines. OX and OY will coincide with the directions of greatest and least stiffness when p = 0, or when tan. 2 *φ =* 0, that is, when Σ*∙ xy ds = 0.*

If, then, the directions of greatest and least stiffness be taken for the axes of *xy* and of *y,* we shall have 2 ∙ *xy ds* = 0, 2 *x2ds =* greatest stiffness = A, 2 *y2ds* =∑ least stiffness = B. These being once known, the stiffness in any other direction, as well as the deflecting tendency, can readily be obtained. Putting P and Q respectively for these quantities, we have

P = A cos. *φ2* + B sin. p2,

Q = 1/2 (B— A ) sin. 2 φ.

The deflecting tendency is thus greatest when *φ =* 45°, that is, when the actual direction of flexure is equally inclined to the directions of greatest and least stiffness. In this case P1 = 1/2 (A+B), Q = 1/2(B-A).

Galileo, who was the first to investigate the law of transverse strain, conceived the lower edge of the beam to be the fulcrum, and each fibre to be exerting its whole strength ; Professor Robison, in the former editions of this work, corrected the supposition in the case of rectangular beams : the above investigation extends it to beams of all forms. (c. E.)

We must now remark, that this correction of the Gali­lean hypothesis of equal forces was suggested by the bend­ing which is observed in all bodies which are strained trans­versely. Because they are bent, the fibres on the convex side have been extended. We cannot say in what propor­tion this obtains in the different fibres. Our most distinct notions of the internal equilibrium between the particles render it highly probable that their extension is propor­tional to their distance from that fibre which retains its for­mer dimensions. But by whatever law this is regulated, we see plainly that the actions of the stretched fibres must follow the proportions of some function of this distance, and that therefore the relative strength of a beam is in all cases susceptible of mathematical determination.