We also see an intimate connection between the strain and the curvature. This suggested to the celebrated James Bernoulli the problem of the *elastic curve, i. e.* the curve into which an extensible rigid body will be bent by a trans­verse strain. His solution in the *Acta Eruditorum* 1694 and 1695, is a very beautiful specimen of mathematical discus­sion, and we recommend it to the perusal of the curious reader. He will find it very perspicuously treated in the first volume of his works, published after his death, where the wide steps which he had taken in his investigation are explained so as to be easily comprehended. His nephew, Daniel Bernoulli, has given an elegant abridgement in the Petersburg Memoirs for 1729. The problem is too intricate to be fully discussed in a work like ours, but it is also too in­timately connected with our present subject to be entirely omitted. We must content ourselves with showing the lead­ing mechanical properties of this curve, from which the ma­thematician may deduce all its geometrical properties.

When a bar of uniform depth and breadth, and of a given length, is bent into an arch of a circle, the extension of the outer fibres is proportional to the curvature ; for, because the curves formed by the inner and outer sides of the beam are similar, the circumferences are as the radii, and the radius of the inner circle is to the difference of the radii as the length of the inner circumference is to the difference of the circumferences. The difference of the radii is the depth of the beam, the difference of the circumferences is the extension of the outer fibres, and the inner circum­ference is supposed to be the primitive length of the beam. Now the second and third quantities of the above analogy, viz. the depth and length of the beam, are constant quan­tities, as is also their product. Therefore the product of the inner radius and the extension of the outer fibre is also a constant quantity, and the whole extension of the outer fibre is inversely as the radius of curvature, or is di­rectly as the curvature of the beam.

The mathematical reader will readily see, that into what­ever curve the elastic bar is bent, the whole extension of the outer fibre is equal to the length of a similar curve having the same proportion to the thickness of the beam that the length of the beam has to the radius of curvature.

Now let ADCB (fig. 5) be such a rod of uniform breadth and thickness, firmly fix­ed in a vertical position, and bent into a curve AEFB by a weight W suspended at B, and of such magnitude that the extremity B has its tan­gent perpendicular to the action of the weight, or parallel to. the horizon. Suppose, too, that the ex­tensions are proportional to the extending forces. From any two points E and F draw the horizontal ordinates EG, FH. It is evident that the exterior fibres of the sections Ee and F*f* are stretched by forces which are in the proportion of EG to FH (these being the long arms of the levers, and the equal thicknesses Ee, F*f* being the short arms). Therefore (by the hypothesis) their extensions are in the same proportion. But because the extensions are proportional to some similar func­tions of the distance from the axes of fracture E and F, the extension of any fibre in the section Ee is to the contem­poraneous extension of the similarly situated fibre in the section *Ff,* as the extension of the exterior fibre in the sec­tion Ee is to the extension of the exterior fibre in the sec­tion *Ff:* therefore the whole extension of Ee is to the whole extension of *Ff* as EG to FH, and EG is to FH as the cur­vature in E to the curvature in F.

Here let it be remarked, that this proportionality of the curvature to the extension of the fibres is not limited to the hypothesis of the proportionality of the extensions to the extending forces: it follows from the extension in the dif­ferent sections being as some similar function of the dis­tance from the axis offracture ; an assumption which can­not be refused.

This, then, is the fundamental property of the elastic curve, from which its equation, or relation between the abscissa and ordinate, may be deduced in the usual forms, and all its other geometrical properties. These are foreign to our purpose ; and we shall notice only such properties as have an immediate relation to the strain and strength of the dif­ferent parts of a flexible body, and which in particular serve to explain some difficulties in the valuable experiments of Buffon on the Strength of Beams.

We observe, in the first place, that the elastic curve can­not be a circle, but is gradually more incurvated as it recedes from the point of application B of the straining forces. At B it has no curvature; anil if the bar were extended beyond B there would be no curvature there. In like man­ner, when a beam is supported at the ends and loaded in the middle, the curvature is greatest in the middle ; but at the props, or beyond them, if the beam extend farther, there is no curvature. Therefore, when a beam projecting twenty feet from a wall is bent to a certain curvature at the wall by a weight suspended at the end, and a beam of the same size projecting twenty feet is bent to the very same curvature at the wall by a greater weight at ten feet distance, the figure and the mechanical state of the beam in the vicinity of the wall is different in these two cases though the curvature at the very wall is the same in both. In the first case every part of the beam is incurvated ; in the second, all beyond the ten feet is without curvature. In the first experiment the curvature at the distance of five feet from the wall is three fourths of the curvature at the wall ; in the second, the curvature at the same place is but one half of that at the wall. This must weaken the long beam in this whole interval of five feet, because the greater curvature is the result of a greater extension of the fibres.

In the next place we may remark, that there is a certain determinate curvature for every beam, which cannot be ex­ceeded without breaking it ; for there is a certain separa­tion of two adjoining particles that puts an end to their co­hesion. A fibre can therefore be extended only a certain proportion of its length. The ultimate extension of the outer fibres must bear a certain determinate proportion to its length, and this proportion is the same with that of the thickness (or what we have hitherto called the depth) to the radius of ultimate curvature, which is therefore determinate.

A beam of uniform breadth and depth is therefore most incurvated where the strain is greatest, and will break in the most incurvated part. But by changing its form, so as to make the strength of its different sections in the ratio of the strain, it is evident that the curvature may be the same throughout, or may be made to vary according to any law. This is a remark worthy of the attention of the watchmaker. The most delicate problem in practical mechanics is so to taper the balance-spring of a watch that its wide and narrow vibrations may be isochronous. Hooke's principle *ut tensio sic vis* is not sufficient when we take the *inertia* and motion of the spring itself into the account. The figure into which it bends and unbends has also an influence. Our readers will take notice that the artist aims at an accuracy which will not admit an error of 1/86400th and that Har­rison and Arnold have actually attained it in several in­stances. The taper of a spring is at present a nostrum in the hands of each artist, and he is careful not to impart its secret Again, since the depth of the beam is thus proportional to the radius of ultimate curvature, this ultimate or break­ing curvature is inversely as the depth. It may be expressed by 1/*d.*