When a weight is hung on the end of a prismatic beam, the curvature is nearly as the weight and the length di­rectly, and as the breadth and the cube of the depth inversely ; for the strength is = ∕'θy∙ Let us suppose that this produces the ultimate curvature Now let the beam be loaded with a smaller weight *w,* and let the curvature 0^7" 1

produced be C; we have this analogy,/*'.w~-∙.* C, and C = ⅛⅛. It is evident that this is also true of a beam supported at the ends and loaded between the props ; and we see how to determine the curvature in its different parts, whether arising from the load, or from its own weight, or from both.

When a beam is thus loaded at the end or middle, the loaded point is pulled down, and the space through which it is drawn may be called the *deflection.* This may be con­sidered as the subtense of the angle of contact, or as the versed sine of the arch into which the beam is bent, and is therefore as the curvature when the length of the arches is given (the flexure being moderate), and as the square of the length of the arch when the curvature is given. The deflection therefore is as the curvature and as the square of the length of the arch jointly; that is, as *×* or as

The deflection from the primitive shape is therefore as the bending weight and the cube of the length directly, and as the breadth and cube of the depth inversely.

In beams just ready to break, the curvature is as the depth inversely, and the deflection is as the square of the length divided by the depth ; for the ultimate curvature at the breaking part is the same whatever is the length ; and in this case the deflection is as the square of the iength.

We have been the more particular in our consideration of this subject, because the resulting theorems afford us the finest methods of examining the laws of corpuscular action, that is, for discovering the variation of the force of cohesion by a change of distance. It is true it is not the atomical law, or *hylαrchic principle* as it may justly be called, which is thus made accessible, but the specific law of the particles of the substance or kind of matter under examination. But even this is a very great point ; and co­incidences in this respect among the different kinds of mat­ter are of great moment. We may thus learn the nature of the corpuscular action of different substances, and per­haps approach to a discovery of the *mechanism* of chemical affinities. For that chemical actions are insensible cases of local motion is undeniable, and local motion is the province of mechanical discussion ; nay, we see that these hidden changes are produced by mechanical forces in many im­portant cases, for we see them promoted or prevented by means purely mechanical. The conversion of bodies into elastic vapour by heat can at all times be prevented by a *sufficient* external pressure. A strong solution of Glauber’s salts will congeal in an instant by agitation, giving out its latent heat ; and it will remain fluid for ever, and retain its latent heat in a close vessel which it completely fills. Even water will by such treatment freeze in an instant by agita­tion, or remain fluid for ever by confinement. We know that heat is produced or extricated by friction, that certain compounds of gold or silver with saline matters explode with irresistible violence by the smallest pressure or agita­tion. Such facts should rouse the mathematical philoso­pher, and excite him to follow out the conjectures of the illustrious Newton, encouraged by the ingenious attempts of Boscovich ; and the proper beginning of this study is to at­

tend to the laws of attraction and repulsion exerted by the particles of cohering bodies, discoverable by experiments made on their actual extensions and compressions. The experiments of simple extensions and compressions are quite insufficient, because the total stretching of a wire is so small a quantity, that the mistake of the 1000th part of an inch occasions an irregularity which deranges any progression so as to make it useless. But by the bending of bodies a distention of 1/100th of an inch may be easily magnified in the deflection of the spring ten thousand times. We know that the investigation is intricate and difficult, but not be­yond the reach of our present mathematical attainments ; and it will give very fine opportunities of employing all the address of analysis. In the 17th century and the beginning of the 18th this was a sufficient excitement to the first ge­niuses of Europe. The cycloid, the catenaria, the elastic curve, the velaria, the caustics, were reckoned an abundant recompense for much study ; and James Bernoulli request­ed, as an honourable monument, that the logarithmic spiral might be inscribed on his tombstone. The reward for the study to which we now presume to incite the mathemati­cians is the almost unlimited extension of natural science, important in every particular branch. To go no further than our present subject, a great deal of important practi­cal knowledge respecting the strength of bodies is derived from the single observation, that in the moderate extensions which happen before the parts are overstrained, the forces are nearly in the proportion of the extensions or separations of the particles. To return to our subject.

James Bernoulli, in his second dissertation on the elastic curve, calls in question this law, and accommodates his in­vestigation to any hypothesis concerning the relation of the forces and extensions. He relates some experiments of lute­strings where the relation was considerably different. Strings of three feet long,

stretched by 2, 4, 6, 8, 10 pounds,

were lengthened 9, 17, 23, 27, 30 lines.

But this is a most exceptionable form of the experiment. The strings were twisted, and the mechanism of the exten­sions is here exceedingly complicated, combined with com­pressions and with transverse twists, &c. We made expe­riments on fine slips of the gum caoutchouc, and on the juice of the berries of the white bryony, of which a single grain will draw to a thread of two feet long, and again return in­to a perfectly round sphere. We measured the diameter of the thread by a microscope with a micrometer, and thus could tell in every state of extension the proportional num­ber of particles in the sections. We found, that through the whole range in which the distance of the particles was changed in the proportion of thirteen to one, the extensions did not *sensibly* deviate from the proportion of the forces. The same thing was observed in the caoutchouc as long as it perfectly recovered its first dimensions. And it is on the authority of these experiments that we presume to announce this as a law of nature.

Dr Robert Hooke was undoubtedly the first who attend­ed to this subject, and assumed this as a law of nature. Mariotte indeed was the first who expressly used it for de­termining the strength of beams : this he did about the year 1679, correcting the simple theory of Galileo. Leibnitz, indeed, in his dissertation in the *Acta Eruditorum* 1684, *De Resistentia Solidorum,* introduces this consideration, and wishes to be considered as the discoverer ; and he is always acknowledged as such by the Bernoullis, and others who ad­hered to his peculiar doctrines. But Mariotte had published the doctrine in the most express terms long before ; and Bulfinger, in the *Comment. Petropol.* 1729, completely vin­dicates his claim. But Hooke was unquestionably the discoverer of this law. It made the foundation of his theory of springs, announced to the Royal Society about the year 1661, and read in 1666. On this occasion he mentions