AC’. In the next place, the solid generated by aAC is two thirds of the cylinder, because the cone generated by cCa is one third of it.

We may now suppose the cylinder twisted till the par­ticles in the external circumference lose their cohesion. There can be no doubt that it will now be wrenched asun­der, all the inner circles yielding in succession. Thus we obtain one useful information, viz. that a body of homogene­ous texture resists a *simple twist* with two thirds of the force with which it resists an attempt to force one part laterally from the other, or with one-third part of the force which will cut it asunder by a square-edged tool ; for to drive a square-edged tool through a piece of lead, for instance, is the same as forcing a piece of the lead as thick as the tool laterally away from the two pieces on each side of the tool. Experiments of this kind do not seem difficult, and they would give us very useful information.

When two cylinders AHK and BNO are wrenched asun­der, we must conclude that the external particles of each are just put beyond their limits of cohesion, are equally ex­tended, and are exerting equal forces. Hence it follows, that in the instant of fracture the sum-total of the forces actually exerted are as the squares of the diameters.

For drawing the diagonal C*e*, it is plain that Ee = A*a* expresses the distension of the circumference ELM, and that the solid generated by the triangle CEe expresses the cohesion exerted by the surface of the circle ELM, when the particles in the circumference suffer the extension Ee equal to A*a*. Now the solids generated by CA*a* and CEe being respectively two thirds of the corresponding cylinders, are as the squares of the diameters.

Having thus ascertained the real strength of the section, and its relation to its absolute lateral strength, let us exa­mine its strength relative to the external force employed to break it. This examination is very simple in the case un­der consideration. The straining force must act by some lever, and the cohesion must oppose it by acting on some other lever. The centre of the section may be the neutral point, whose position is not disturbed.

Let F be the force exerted laterally by an exterior particle. Let *a* be the radius of the cylinder, and *x* the indeter­minate distance of any circumference, and *dx* the indefinitely small interval between the concentric arches ; that is, let *dx* be the breadth of a ring and *x* its radius. The forces being as the extensions, and the extensions as the distances from the axis, the cohesion actually exerted at any part of any ring will be *fxdx/a*. The force exerted by the whole ring (being as the circumference or as the radius) will be

*, t∕,t*

*~~f~~* The momentum of cohesion of a ring, being as

*nc^dx*

the force multiplied by its lever, will be*f~^~∙* The accu-

*xidx* mulated momentum will be the sum or fluent off ;

*a ai*

that is, when *x = a, it* will be }/— = *if,t,∙*

Hence we learn that the strength of an axle, by which it resists being wrenched asunder by a force acting at a given distance from the axis, is as the cube of its diameter.

But, further, *⅛fa3* is =∕α, × J *a.* Now *fa2* represents the full lateral cohesion of the section. The momentum therefore is the same as if the full lateral cohesion were ac­cumulated at a point distant from the axis by one fourth of the radius, or one eighth of the diameter of the cylinder.

Therefore let F be the number of pounds which measure the lateral cohesion of a circular inch, *d* the diameter of the cylinder in inches, and *l* the length of the lever by which

the straining force *p* is supposed to act ; we shall have F *×⅜d3=pZ,andF≤=p.*

We see in general that the strength of an axle, by which it resists being wrenched asunder by twisting, is as the cube of its diameter.

We see also that the internal parts are not acting so powerfully as the external. If a hole be bored out of the axle of half its diameter, the strength is diminished only one eighth, while the quantity of matter is diminished one fourth. Therefore hollow axles are stronger than solid ones con­taining the same quantity of matter. Thus let the diameter be 5, and that of the hollow 4 ; then the diameter of an­other solid cylinder having the same quantity of matter with the tube is 3. The strength of the solid cylinder of the diameter 5 may be expressed by 53, or 125. Of this the internal part (of the diameter 4) exerts 64 ; therefore the strength of the tube is 125—64 = 61. But the strength of the solid axle of the same quantity of matter and dia­meter 3 is 33, or 27, which is not half of that of the tube.

Engineers, therefore, have of late introduced this im­provement in their machines, and the axles of cast iron are all made hollow when their size will admit of it. They have the additional advantage of being much stiffer, and of afford­ing much better fixture for the flanches which are used for connecting them with the wheels or levers by which they are turned and strained. The superiority of strength of hollow tubes over solid cylinders is much greater in this kind of strain than in the former or transverse. In this last case the strength of this tube would be to that of the solid cylinder of equal weight as 61 to 32 and a half nearly.

The apparatus which we mentioned on a former occasion for trying the lateral strength of a square inch of solid mat­ter, enabled us to try this theory of twist with all desirable accuracy. The bar which hung down from the pin in the former trials was now placed in a horizontal position, and loaded with a weight at the extremity. Thus it acted as a powerful lever, and enabled us to wrench asunder speci­mens of the strongest materials. We found the results perfectly conformable to the theory, in as far as it deter­mined the proportional strength of different sizes and forms ; but we found the ratio of the resistance to twisting to the simple lateral resistance considerably different, and it was some time before we discovered the cause.

We had here taken the simplest view that is possible of the action of cohesion in resisting a twist. It is frequently exerted in a very different way. When, for instance, an iron axle is joined to a wooden one by being driven into one end of it, the extensions of the different circles of particles are in a very different proportion. A little consideration will show that the particles in immediate contact with the iron axle are in a state of violent extension; so are the particles of the exterior surface of the wooden part, and the intermediate parts are less strained. It is almost impossible to assign the exact proportion of the cohesive forces exert­ed in the different parts. Numberless cases can be point­ed out where parts of the axle are in a state of compression, and where it is still more difficult to determine the state of the other particles. We must content ourselves with the deductions made from this simple case, which is fortunate­ly the most common. In the experiments just now men­tioned, the centre of the circle is by no means the neutral point, and it is very difficult to ascertain its place ; but when this consideration occurred to us, we easily freed the experiments from this uncertainty, by extending the lever to both sides, and by means of a pulley applied equal force to each arm, acting in opposite directions. Thus the centre became the neutral point, and the resistance to twist was found to be two thirds of the simple lateral strength.

We beg leave to mention here, that our success in these