also been measured. The labour and ingenuity exerted in these operations have been very great.

As, generally speaking, all bodies expand by heat and contract by cold, a rod or chain varies in absolute length as it varies in temperature. A steel rod at the tempera­ture of water boiling is the one thousandth part longer than at the temperature of water freezing. Hence, when great accuracy is required, the temperature of the measure must be noted, that its precise length may be known.

When the sides of the primary triangles are known, they serve as bases by which the situation of objects near them may be determined. In this way, the position of any num­ber of points in a road, or the course of a river, also the situation of towns, country-seats, the boundaries of enclo­sures, and every feature of a country, may be found, and laid down in a plan or map : and to construct it with accu­racy, the principal points should be laid down by their cal­culated distances from each other, rather than by the mea­sures of the angles formed by the lines joining them, because a point may be more correctly laid down by the intersec­tion of two arcs of circles, than by that of lines which make given angles with a third line.

A small extent of the earth’s surface may be regarded as a plane, and lines perpendicular to it as parallel to one an­other. However, in an extensive survey, such as that of Britain, the curvature of the earth must be taken into ac­count, and then its figure and magnitude enter as elements into all the calculations.

This connection between the figure of the earth and the magnitude and position of lines traced on its surface affords, reversely, the means of determining the former when the latter are known : so that such surveys, besides their immediate object, are applicable to the solution of the still more sublime problem of finding the magnitude and figure of the earth itself.

*Of Terrestrial Refraction.*

It is a matter of experience, that the rays of light pro­ceeding from the heavenly bodies are bent from their original rectilineal direction in passing through the atmo­sphere. A ray in its progress may be considered as passing through a very great number of thin *strata* of air, which are denser the nearer they are to the earth ; and these, by their action on the particles of light, bend the ray downwards, so that, in fact, its path is a curve line concave to­wards the earth, and situated in a vertical plane passing through the luminous object and the eye of the observer.

To understand the nature of this effect, let AB (fig. 1) represent a portion of the earth’s surface, and DFG the upper boundary of the atmo­sphere : a succession of par­ticles of light from the sun or a star S, proceed on a straight line, until they arrive at F ; afterwards, in their passage through the atmo­sphere, their path is gradually bent into the curve FE, so that when they reach the eye at E, the object S appears as if it were at S', in the direction of a straight line which touches the curve at E.

This bending of the ray is called *refraction ;* it increases the apparent elevation of the heavenly bodies above the horizon, except when they are in the zenith, and in that position there is no refraction.

The rays of light, by which terrestrial objects are render­ed visible, are, in their passage through the atmosphere, bent downwards, exactly as those proceeding from the heavenly bodies ; so that the apparent elevation of a re­mote object is always greater than its true elevation. This incurvation is called *terrestrial refraction ;* in the case of the heavenly bodies, it is *astronomical* refraction.

Terrestrial refraction varies with the state of the atmo­sphere, so that an object appears more elevated at one time than at another : the displacement, however, is always in a vertical plane, but never sensibly in a horizontal direc­tion. How it may be found for any case will appear from the following problem.

Prob. I.

Having given the apparent positions of two remote stations, as seen from each other *ar the same instant,* to determine the error produced by refraction.

Let A and B be the stations (fig. 2), and C the earth’s centre ; draw the lines CA, CB, and produce them towards Z and V, the zeniths of the stations ; join AB ; then the true zenith distance of B, as seen from A, is the angle ZAB, and the true zenith distance of A, as seen from B, is the angle VBA. These, however, cannot be directly measured, for, by refraction, the point B, as seen from A, appears elevated to the posi­tion *b;* and the point A, as seen from B, is elevated to the position *a.* The errors, then, produced by refraction are the angles *b*AB and *a*BA, and these will be nearly equal, if the angles be observed at the same instant, which may be done by setting two watches to the same time, or mak­ing a signal at one station so as to be seen from the other.

Put the greater apparent zenith distance ZA*b* = *d,* the lesser VB*a* — *d',*

the refraction *b*AB = *a*BA *= r,*

the angle C at the earth’s centre = C.

Then ZAB = ABC + C, and VBA = BAC + C (Geom. 23, 1), therefore ZAB + VBA = ABC + BAC + C + C.

But ABC + BAC + C = 180°,  
therefore ZAB + VBA = 180° -∣- C.

Again, ZAB = ΖΑ*b* + *b*AB *= d + r,* VBA = VB*a* + *a*BA *= d'+r,* therefore ZAB + VBA = *d* + *d'* + 2r.

Put the two values of ZAB + VBA equal to each other, and we have

*d + d* + 2r = 180° + C, and 2r = 180° + C—*(d + d),* and *r —* 90° 4+ 1/2C — 1/2 *(d* + *d).*

*Ex.* In the British survey, on Wisp Hill and Cross Fell are two stations in a triangle, which connects the north of England with the borders of Scotland. Their distance is computed at 235,018∙6 feet — 44∙511 miles, this corresponds on the surface of the earth to an arc of 38' 33''∙7. From Cross Fell, Wisp Hill was seen depressed 30' 48" below the horizon, and from the latter place the former was found to have a depression of 2' 31''. Here we have

*d* = 90° 30' 48'', *d'* = 90° 2' 31", C = 38' 33"∙7 ; hence  
*r =* 90° + 1/2C — 1/2 *(d + d')* = 2' 37"∙3.

ln this case the error produced by refraction is nearly 1/15th of the arc intercepted between the stations. Dr Mas- kelyne reckoned it to be 1/10th. Delambre and Legendre, French mathematicians, estimated it, the former at 1/14th, and the latter at 1/11th ; and Col. Mudge, by numerous and correct observations, found its medium value to be about 1/12th.