Because

C—O(=BCA-BOA) = BHA—BOA (Geom.24, 1), and HAO = BHA —BOA,

therefore C — O = HAO.

In the triangle OHC, sin.CHO or CHB : sin.HCO: :CO: HO ; and in the triangle OHA, HA : HO : : sin. HO A : sin. HAO. Remarking now that CHB ≡ CAB = A, and that HCO = CHB — COB = A — *v,* also that HA and CA = *m* are nearly equal, these proportions may be expressed thus : sin. A: sin.(A — *v) : : d* : HO, *m* : HO : : sin. O : sin. HAO ;

hence HO = </sin-.(A~t,)> sin. A

, . TT,„ HO × sin. O dsin.(A— t>)sin. O and sin. HAO = = i—:—( .

*m m* sin. A

But, on account of the smallness of the angle HAO, the . - . . . ., . sin. HAO ,

number of minutes it contains will be —:—∏— nearly; sin. Γ

therefore, because C — O = HAO, we have, in minutes of a degree,

r, rκ rZsin. (A—») sin. O V√ — v√ — ; ∙. . XJ ■

msιn. A sin. Γ

This expression for the difference of the angles C and O is not quite so accurate as the former, yet, in practice, it is near enough the truth. It requires that approximate values of the distance AC and the angle BAC be known ; the distance *d* should, however, be accurately determined.

It is obvious that if the instrument were placed at H, in the circumference of a circle passing through A, B, C, the observed angle AHB would then be equal to the angle C at the station. This may be done by moving it along OB until the angle CHB is found to be equal to CAB, and then no correction is wanted.

*Ex.* In a survey, A, B, C are three stations, and AB = 6196 feet. At A, the angle BAC = 63° 44', and at O, a point 11∙5 feet from the centre of the station C, the angle AOB = 45° 42' ; the angle COB = 50° 18'. Hence it is required to find the angle ACB.

The angle ACB will be nearly equal to AOB ≡ 45° 42', hence CBA = 70° 34' nearly. In the triangle CAB we have now approximate values of all the angles, and a side AB ; hence (by the first case of oblique-angled triangles) BC = 7763, and AC = 8164 feet.

To apply the formulas for reducing to the centre, we have *m =* 8164 O = 45° 42' O + *v* = 96° 0'

*n* = 7763 *v* = 50 18 A—*v*=13 26

*d =* 11·5 A = 63 44

to find C — O. The logarithmic calculation by the first formula may stand thus :

*d* 1∙060698 *d* 1∙060698

sin. (O + *v)* 9∙997614 sin.*v* 9∙886152

*m* ) Ar. Comp. 6·088097 *n* Ar. Comp. 6∙109970 sin. 1' log. 3∙536274 sin. 1' log. 3∙536274

4\*816 0∙682683 3'∙918 0∙593094

Hence C— O = 4'∙816 —3'∙918 = 0'∙898 = 54" nearly ; this gives C = 45° 42' 54".

*calculation by the second formula.*

*d*  1∙060698

sin. (A— *v)*  9∙366075

sin. O 9∙854727

*m* ) 6·088097

sin. A Ar. Comp. log... 0·047331 sin. 1' 3∙536274

C — O = 0'·898 = 54'' — 1∙953202

The correction is positive, because the angle *A — ν* is

less than 180°, and therefore sin. (A —*v*) is a positive quan­tity.

When a theodolite is employed in surveying, the angles are taken at once in *a* horizontal plane ; but when a sex­tant is used, the angles are measured in the planes of the objects, and if they are oblique, the corresponding horizon­tal angles are found by calculation.

Prob. IV.

Having given the inclination of each of two lines to the horizon, and the oblique angle they contain, to find the corresponding horizontal angle.

Let AB, AC be the straight lines, which contain between them thc given oblique angle BAC : in AV, a perpendicu­lar to the horizon, take any point H, and let a horizontal plane pass through II and meet AB, AC in B and C ; join HB, HC, BC ; then BHC is the horizon­tal angle corresponding to the given oblique angle BAC, and HAB, HAC the complements of the inclinations of the lines AB, AC to the horizontal plane. Put the angles HAB = *b*, HAC = c, the given oblique angle BAC = A, and its corre­sponding horizontal angle BHC = H.

Then, supposing AH to be the radius of a circle, and = 1, it is evident that in the right-angled triangles AHB, AHC,

AB = sec. *b*, HB = tan. *b,* AC = sec. *c,* HC = tan. *c.* Now, in the triangles ABC, HBC, by Trigonometry, AB2 + AC2= BC2 + 2AB × AC × cos. A, HB2 + HC2 = BC2 + 2HB × HC × cos. H ;

Hence, by subtracting equals from equals, and observing that AB2 —HB2 = AH2, and AC2 —HC2 = AH2, we have 2AH2 = 2AB× AC × cos. A — 2HB×HC×cos.H, that is, 1 = sec. *b* sec. *c* cos. A — tan. *b* tan. *c* cos. H.

From this expression, after substituting ——, and —-—  σ cos. *b* cos. c

„ , . , sin. *b* , sin. *c . , ,*

for sec. *b* and sec. *c,* also r and tor tan. *b* and

cos. *b* cos. c

tan. *c,* we get

sin. *b* sin. *c* cos. H = cos. A — cos. *b* cos. *c.*

Now cos. H = 1 — 2 sin.2 1/2H (Algebra, 248), hence, by substituting and transposing, we get this other expres­sion,

2 sin. *b* sin. c sin.2 1/2H = cos. *b* cos. *c* + sin. *b* sin. *c —* cos. A. Again, cos. *b* cos. *c* + sin. *b* sin. c = cos. *(b — c)* (239), and cos. (*b* — c) — cos. A = 2 sin. — — sin.

A

(240) ; therefore,

2 x ∙ ∙ sιu o∙ A -L *(b—*c) . A—*(b—*c)

2 sin. *b* sin. *c* sm.2 ⅛H = 2 sin. ~ ' ' sin. -= ∙

A A

τp iA + ⅛ + c A-p(δ — c)

If we put *-—i— = s,* then —i-i⅛ *ι = s — c,*

= ,\_b, and

sin. *b* sin. *c* sin.2 1/2H = sin. *(s — b)* sin. (s —*c),* and hence

. ,ττ /sin. *(s — b)* sin. *(s∙*—∙c) sin. ÀH = *J ;—f-.——-.*

*l \*t* sm. *b* sm. c

Note. In this formula, the sines are supposed to be computed to a rad. = 1, but in the table of log. sines, the rad. is a number of which the log. is 10; therefore, to adapt the formula to the table, we must divide each sine