to rad. = 1 by this number, so that calling the number R, we have

sin. JH = R ∕⅛¾→-t) (1)

2 V sin. o sin. c ' '

When the lines which contain the given oblique angle have their inclinations to the ho­rizontal plane less than 2° or 3°, the general solution does not conveniently apply. In this case, instead of seeking the horizontal angle directly, it is better to find its difference from the oblique angle.

To investigate a rule, let the horizontal triangle BHC (see last figure) be applied upon the plane of the oblique triangle BAC, so that they may still have a common base BC ; join AH, pro­ducing it to K, and draw HP, HQ perpendicular to AB, AC. And because BHK = BAH -∣- ABH, and CHK

— CAH + ACH, therefore BHC = BAC + ABH + ACH, and putting *d for* the difference BHC — BAC, we have

*d* = ABH -J- ACH.

Put *r* for AH, *m* for the angle HAP, and *n* for the angle HAQ; then HP *— r* sin. *m,* HQ = *r* sin. *n,* AP = *r* cos. *m,* AQ = *r* cos. *n,* and, by the formula· of Art. (246) Algebra, HP 4 HQ sin. *m* + sin. *n ,, . , ,*

. ,∖- ■—τ-≡ = ; = tan∙ i(°, + n) = tan. ⅛A,

AQ 4- AP cos. *n* -∣- cos.*m 9' τ 2 β ’*

HP — HQ sin. *m —* sin. *n ,, , . , ,*

V75 in = — = cot∙ J(»> + *n) =* cot. i A ;

AQ — AP cos. *n —* cos. *m 3v , , «*

therefore HP + HQ = (AQ + AP) tan. 1/2A, HP — HQ = (AQ — AP) cot. 1/2A ;

and, by adding and subtracting,

AQ + AI∖ AQ—AP . , .

HP = i—- tan. JA -j 5 cot. 1/2A,

ttλ AQ 4-AP , . AQ—AP HQ = - tan. JA cot. JA.

Supposing *b* and c to denote the same things as in the general solution, put *b'* for the difference between *b* and a right angle, and *d* for the difference between *c* and a right angle, then *b* and *d* are the inclinations of the sides of the given oblique angle to the horizontal plane.

The lines BA, BH are nearly equal, because the one is the secant and the other the tangent of an angle nearly

— 90° ; and the same is true of the lines CA, CH ; hence the points A and H will be near each other, and the angles ABH, ACH will be small, and BP ≡ BH nearly, also CQ — CH nearly ; and since

Z>, *l∕*

BA = sec. *b —* cosec. *b =* Jcot. - -∣- Jtan. - )

i « ( Algebra

BH = tan. *b ~* cot. *b' —* Jcot∙ — Jtan. j ∙

3' therefore, by subtracting, AP = BA — BH = tan. -,

*• . d*

In like manner it appears that AQ = CA — CQ = tan. θ. <w

These values of AP and AQ being substituted in the values of HP and HQ, we have

*t d 1 b d b*

tan. - + tan. - tan. - — tan. -

HP = - tan. JA + - cot. J A,

*d 1 4 b d b'*

Un. - + tan. - tan.^-tan. -

HQ= - - tan. JA cot. JA.

Now, in the triangles HBP, HCQ, because HB = cot. *b*

*— τ* τ⅛ and HC = cot. *d = —-—-,* we have

Un. *b'* tan. *d*

HP

sin. HBP = ⅛⅛ = HP × tan. *b' ;*

**Ho**

HQ

sin. HCQ = = HQ × tan. *c ;*

**HL**

therefore, by adding, and observing that the sum of the sines of the angles HBP, HCQ, is, by reason of their smallness, almost equal to twice the sine of half their sum, that is, to twice the sine of J(H — A) = 1/2d, we have

2 sin. 1/2*d* = HP × tan. *b* + HQ × tan. *c'.*

In this expression, substitute the values already found for HP and HQ, and the result will be J(tan.c'-}-tan.Z>')(tan. -i + tan.^)tan. JA, I 2sin.⅛d=J % 2' (2)

—J(tan. *d—* tan. *b')* (tan.-—Un. - 1 cot. J A. ∣

The approximation will be sufficiently accurate, and more simple, if we put the arcs instead of the sine and tangents, to which they are almost equal. The formula then becomes ∕√ + δ'∖2 1 *rd—b∖t .*

*d=* (—J tan. JA— Ç-g— *j* cot. JA.

In this formula, the arcs or angles *d, b, d* are expressed in parts of the radius. If we suppose the same letters to express the number of minutes in each, then, observing that an arc of 1' = ∙00029888, in parts of the radius, we must substitute d × ·00029888, or, which is the same thing, d/3438

instead of *d,* and similarly, b'/3438 and c'/2428 instead of *b'* and

*d.* This being done, and the quantities tan. 1/2A and cot. 1/2A being divided by R, the radius, for the convenience of lo­garithmic calculation, we have

”= {(Ψ),,⅛δ-(⅛-7⅛δ}s38 <3>

Note. We have supposed the angles *b', d* to be both elevations. If one be a depression, the formula will still hold true, provided the arc of depression be regarded as a negative quantity. This may be inferred from the position of the lines, also from the rules for the signs of the cosine and sine of an arc (Algebra, 225).

*Ex.* 1. The angles of elevation of two straight lines, which contain an oblique angle of 64° 10', are 6° 20' and 8° 46' : Find the corresponding horizontal angle.

In this case A = 64° 10' Log. calcu. by the 1st formula. *b*=90°-6°20'=83 40 sin. *b* . 0∙002659 c = 90 —8 46 =81 14 sin. *c* 0·005104

2)229 4

*s=* 114 32

*t-b=* 30 52 sin. (r — *b)* 9-710153

*s — c =* 33 18 sin. (*s* — *c)* 9∙739590

2)Γ9∙457506 sin. J H 32° 22' 42" 9-728758

The horizontal angle H = 64 45 24, the answer.

As the ar. comp. of two logarithms are added, we ought to reject 20 from the sum, or 10 from half the sum ; but the formula requires that 10, the log. of rad., be added to half the sum. As these operations compensate each other, both are here omitted.

*Ex.* 2. As an example of the second formula, let A = 97° 36'; *d,* an angle of elevation, = l° 30'; *b,* an angle of depression, = 1° 6': Find the horizontal angle H.

Herec,= + 90,, *b* = — 66', *d + b'* = 90'— 66' = 24', *d*

*— b* = 90' + 66' ≡ 156'. The calculation by logarithms may be thus :