Log. Log.

(^g^) 2 158362 (^^) 3∙784190

tan. 1/2A 10∙057777 cot. 1/2A 9∙942223

3438 Ar. Co. 6∙463694 3438 Ar. Co. 6∙463694

0'∙048 — 2∙679833 l'∙549 0·190107

Here 20 is subtracted from each sum, viz. 10 because of the ar. co. of the log. that is added, and 10 because the rad. is a divisor of each term of the formula. The calculation gives *d =* 0'∙048 — 1'∙549 = — 1'∙501 = — 1' 30", a nega­tive quantity, and the required horizontal angle is 97° 36' — 1' 30" = 97° 34' 30”.

Scholium. If a station be taken at a known height above a horizontal plane, and the oblique angles subtended by any number of objects be measured, also their depressions, the corresponding horizontal angles may be found by this prob­lem. Their horizontal distances may also be found, and their places laid down in a map of the country.

When a tract to be surveyed has been covered with a series of triangles, so as to connect the principal points, and all the angles of each, and a side of one, are known, the sides of all the triangles may be found, and a plan made, by con­structing the triangles on the sides of each other ; but in a plan constructed by this method, an error in the position of one triangle must necessarily affect the positions of all the others that depend on it.

To avoid this inconvenience, we may determine the po­sition of *a* side of one of the triangles in respect of a meri­dian, or line drawn due north and south, by a compass, or more accurately by astronomical observations, and then cal­culate the distances of all the stations from the meridian, and also the distances from each other reduced to the di­rection of the meridian : from these, the position of each station may be laid down in the plan independently of the others, and also the direct distance between any two points may be easily found. The manner of proceeding will ap­pear from the following example.

Let A, B, C, D, E, F, be six stations connected by four triangles ABC, BCD, BDE, EDF ; the angles are BAC = 79° 20' CBD = 39° 20' ABC = 51 31 BCD= 69 28 ACB = 49 9 BDC= 71 12

sum =180 sum =180

DBE= 45° 28' EDF= 62° 3' BDE= 72 3 DEF= 52 25

BED= 62 29 DFE = 65 32 sum =180 sum = 180

A side AB of one of the triangles is 4213 yards, and it makes with NS, the meridian, an angle of 62o 52' at the point A. Find the points in which perpendiculars from the sta­tions cut the meridian, and the length of each perpendicular.

Draw B*b*, *Cc, Dd, Ee,* F*f* perpendicular to the meridian, and B*n*, Dy, parallel to it, forming the right-angled tri­angles AB*b*, BC*m*, BD*n*, ED*p*, FD*q*. Because the angles of the four triangles ABC, BCD, BDE, DEF are given, and also AB, a side of one of them, the five lines AB, BC, BD, DE, DF may be found from AB and each other (by case 1, oblique-angled triangles). Their logarithms will be AB 3∙624591, BD 3∙733559, DF 3·578533, BC 3∙738255, DE 3·638690.

In the right-angled triangle AB*b*, the side AB (or its log.) and the angle B *Ab* = 62· 52' are known ; hence we find *Ab* = 1921∙4, Bi = 3749∙3 yards.

If from AB*m*= 117° 8' (the sup. of BA*b*) the angle

ABC = 51° 31' be taken, there remains CB*m* = 65° 37’. Therefore, in the right-angled triangle CBm, the angles and the log. of BC are now known ; hence we find B*m* = 2259∙6, *Cm* = 4985∙2.

And if from CBn = 65° 37', CBD = 39° 20' be sub­tracted, there remains DB*n* = 26° 17'. Therefore, in the right-angled triangle DB*n*, the angles, and the log. of BD, are known ; and hence

Bn = 4854∙7, Dn= 2397∙6.

From BD*q*= 153° 43' (the sup. of BD*n*) subtract BDE= 72° 3', and there remains ED*p*=81° 40'; then, in the right-angled triangle ED*p*, the angles, and the log. of DE, are known, and hence D*p* = 630∙7, E*p* = 4306∙1.

Lastly, subtracting EDF = 62° 31' from ED*p* = 81° 40', there remains FD*q*= 19° 37'; and hence, in the right-angled triangle DF*q*, we find D*q* = 3569∙1, F*q* = 12724.

To determine the stations, we have now

Yard». Yards.

*Ab* - 1921∙4, B*b* =3749∙3,

Ac= *Ab* + B*m* = 4181·0, *Cc = Cm —* B*b* = 1235∙9,

*Ad— Ab* + B*n* = 6776∙1, *Dd=* B*b* —D*n*= 1351·7,

*Ae= Ad* +D*p* = 7406∙8, E*e* = D*d* +Ep = 5657∙8,

A*f*= *Ad* + D*q* = 10345∙2, F*f* = D*d* +F*q* = 2623∙8.

By these numbers the position of each station may be laid down with great accuracy in a plan, independently of the others : also the distance between any two may be readily found ; for example,

CE = √ (Cc + Ee)2 + (Ae— Ac)2 (Geom. 13, 2).

*Of Surveying with a compass.*

When a tract of country is to be surveyed with expedi­tion, and no great degree of accuracy is required, the com­pass, notwithstanding its imperfections, is preferable to any other instrument. Even in a correct survey, it may be ap­plied with advantage in filling up the less important parts of a plan.

The principle which renders the compass applicable to surveying is generally known. The instrument should ad­mit of being fixed in a horizontal position on the top of a staff, and it should be furnished with two sights diametri­cally opposite to each other. The circumference of a circle immediately under or opposite to the point of the needle should be divided into degrees, and it may be numbered both ways, beginning from the extremities of the diameter that passes through the sights ; or it may be numbered all one way from 0o to 360°, beginning at a point immediately under one of the sights.

The use of the instrument is to determine the bearings of objects as seen from each other, or the angles which the lines joining them make with the magnetic meridian. Thus, if the perimeter of a polygon is to be surveyed and laid down on paper, the instrument must be placed at one of its angles, so that an object at the next angle may be seen through the two sights. The needle of the compass being then allowed to turn freely on its pivot, it will settle in the magnetic meridian, and point out on the divided circle the number of degrees the line deviates from thc north or south and on which side of the meridian it lies. If this be done at each angle, the positions of all the lines in respect of the magnetic meridians passing through their intersections will be determined. The sides of the polygon may be mea­sured in passing from one angle to another ; and then, as thc meridians may be supposed parallels, sufficient data will be obtained for constructing a plan of the figure, and veri­fying the accuracy of the survey ; for any angle of the figure will manifestly be equal to the sum or difference of the angles which the lines that contain it make with the meridian passing through their intersection, according as they lie on opposite sides or the same side of that meridian.