two refracting substances, C the centre, V the vertex, AV the semi-aperture, AP its sine, PV its versed sine, and F the focus of parallel rays infinitely near to the axis. Let the extreme ray *a*A, parallel to the axis, be refracted into AG, crossing CF in *f,* which is therefore the focus of ex­treme parallel rays.

*The rectangle of the sine of incidence, by the difference of the sines of incidence and refraction, is to the square of the sine of refraction, as the versed sine of the semi-aperture is to the longitudinal aberration of the extreme rags.*

Call the sine of incidence *i*, the sine of refraction *r*, and their difference *d.*

Join CA, and about the centre *f* describe the arch AD.

The angle ACV is equal to the angle of incidence, and CA*f* is the angle of refraction. Then, since the sine of in­cidence is to the sine of refraction as VF to CF, or as A*f* to *Cf,* that is, as D*f* to *Cf,* we have

CF : FV = C*f*:*f*D,

by conversion CF : CV = *Cf* : CD, altem, conver. CF — *Cf:* CV — CD = CF : CV or F*f*: VD = CF ; *CV = r : d.*

.τ AP2 \_ AP2 . , AP3

Now PV cp + cv \_ 2cv nearly, and PD

AP3 AP\*

= gjτy nearly, = nearly. Therefore PV : PD = FV ; CV, and DV : PV = CF : FV nearly.

We had above F*f*: VD = *r* : *d,*

and now VD : PV = CF : FV ≡ *r : i ;*

therefore F*f*: PV = *r2 : di,*

**and F*f* =^∙PV. *Q.E.D.***

The aberration will be different according as the refrac­tion is made towards or from the perpendicular ; that is, according as *r* is less or greater than *i.* They are in the Γ' it

ratio of — to -7-, or of *r*3 to *i*3. The aberration therefore *dι dr*

is always much diminished when the refraction is made from a rare into a dense medium. The proportion of the sines for air and glass is nearly that of 3 to 2. When the light is refracted into the glass, the aberration is nearly 4/3 of PV ; and when the light passes out of glass into air, it is about 9/2 of PV.

rι AP3 *r3*

*cor.* 1. F*f = ^j∙' 2lfy* nearly, and it is also =

**AP- AP\***

' sfV, because PV = 2CV nearly, an^ \*·^= PV : ^v. *Cor.* 2. Because *fP* : PA = F*f*: FG

or FV: AV = F*f*: FG nearly,

AV we have FG, the lateral aberration, = *Ff × =*

r\* \_ AV3 \_ r5 AV3 *tP* 2FV\* “ V 2CV\*'

FG *cor.* 3. Because the angle FA*f* is proportional to FG/FV

very nearly, we have the angular aberration FA*f* — —

AV3 \_ r> AV\*

X 2FVj ~ *it ’* 2CV3'

In general, the longitudinal aberrations from the focus of central parallel rays are as the squares of the apertures di­rectly, and as the focal distances inversely ; and the lateral aberrations are as the cubes of the apertures directly, and the squares of the focal distances inversely ; and the angu­lar aberrations are as the cubes of the aperture directly, and the cubes of the focal distances inversely.

The reader must have observed, that to simplify the in­vestigation, some small errors are admitted. PV and PD are not in the exact proportion that we assumed them, nor is *Df* equal to F V. But in the small apertures which suf­fice for optical instruments, these errors may be disre­garded.

This spherical aberration produces an indistinctness of vision, in the same manner as the chromatic aberration does, viz. by spreading out every mathematical point of the object into a little spot in its picture ; which spots, by mixing with each other, confuse the whole. We must now determine the diameter of the circle of diffusion, as we did in the case of chromatic dispersion.

Let a ray *βα* (fig. 6) be refracted on the other side of the axis, into *a*H*φ,* cutting *afG* in H, and draw the perpen­dicular EH. Put AV = *a,* αV = a, *Nf* (or VF or Vp, which in this comparison may be taken as equal) *— f, Ff = b,* and *f*E = *x.*

' AV2 : *α*V2= *Ff* : Fp (already demonstrated), and F*φ* αs, - „. p f . , a2 a2Z> — *ab b*

*= —b,* and *Ff— Ff* (or∕p) = δ — — ά = -i = —

(*a2* — *α2*) = *(a + α) (a—a).* Also P*f:* PA = *fE:* EH, or *f: α =: x : -y =:* EH. And Pπ : *Pφ* = EH : Eφ or *α :f ax ax „ ∙vι c c ax . σx + ax*

— — : — = Ep. 1 herefore *ff — —* + x ≡

ip *b*

*= ~(a + a).* Therefore - ∙ (a -f- a) = —. (a 4- a) *(a—a),*

and - = — (a — a), and *x = —5 a fa — a).* Therefore *x a ai di j*

is greatest when *a (a—*α) is greatest ; that is, when α *=* 1/2*a.* Therefore EH is greatest when Pπ is equal to the half of AP. When this is the case, we have at the same time

— ∙ α (α — a) = ∣i ∙∣ a!, and *x = ∣ b,* or EH = ∣ FG. That is, the diameter of the circle of aberration through which the whole of the refracted light must pass, is 1/4 of the diameter of the circle of aberration at the focus of parallel central rays. In the chromatic aberration it was 1/2 ; so that in this respect the spherical aberration does not create so great confusion as the chromatic.

We are now able to compare them, since we have the measure of both the circles of aberration.

It has not been found possible to give more than four inches of aperture to an object-glass of 100 feet focal dis­tance, so as to preserve sufficient distinctness. If we com­pute the diameter of the circle EH corresponding to this