lenses, so that DL is the ultimate focal distance, reckoned from the last surface.

It is plain that DL may be determined by means of *a', bl, m', pl,* and CI in the same manner that BI was deter­mined by means of *a*, *b, m, p,* and AG.

*fi*

The value of BI is *f — mα l- — ft q∙* Take from this P

y∙2

the interval 3, and we have CI *—f — ma, ~ — i —fi q.*

*fl*

Let the small part — *ma —, —* 3 — *f2 q* be neglected for the present, and let CI be supposed = *f:* As we formed *φ, f,* and *q,* by means of *a*, *b, m, n,* and *r*, let us now form *φ',f',* and *q',* for the second lens, by means of *a, b', m', n'* (= 1/*a'* - 1/*b'*), and *r'*; then *φ'* will be the focal distance of a slender pencil refracted by the first surface, *f'* will be the focal distance of this pencil after two refractions, and *q'* will be the co-efficient of the aberration, neglecting the thickness and interval of the lenses.

Proceeding in this way, DL will be = *f'— mβ '-~' —friq'∙*

/» But because CI is really less than *f* by the quantity *ma —*

+ δ + *f2q',* we must (by Lemma 3) subtract the product DL *fn∖*

of this quantity, multiplied by ∣,-p (which is nearly∙yτj,from ∕'-^∕V

By this process we shall have

dl=∕∙-∕-(=∙+a + ^)-∕¾ + λ

The first term *f'* of this value of DI is the focal dis­tance of a slender pencil of central rays refracted by both lenses, neglecting their thickness and distance ; the second ∕mβ 1 3 *1 n∣'∣3∖ . ,* term, —*fκ , ~r -p ^r ~p~J>* ιs the correction necessary

for these circumstances ; and the third term, —*f2(q* + *q'*), is the correction for the aperture 2e. And it is evident that *q'* is a formula precisely similar to *q,* containing the same number of terms, and differing only by the *m', a', n',* and *r,,* employed in place of *m, a, n,* and *r*.

It is also evident, that if there be *a* third lens, we shall obtain its focal distance by a process precisely similar to that by which we obtained DL ; and so on for any number of lenses.

Thus have we obtained formulæ by which the foci of rays are determined in the most general terms, and in such a manner as shall point out the connection of the curvatures, thicknesses, and distances of the lenses, with their spherical aberrations, and with the final aberration of the compound lens, and give the aberrations in separate symbols, so that we can treat them by themselves, and subject them to any conditions which may enable us to correct one of them by another.

We also see in general, that the corrections for the thick- ness and distance of the lenses are exhibited in terms which involve only the focal distances of central rays, and have very little influence on the aberrations, and still less on the ratio of the aberrations, of the different lenses. This is a most convenient circumstance ; for we may neglect them while we are determining *q* and *q',* and in determining the ratio of the focal distances of the several lenses, on which the correction of the chromatic aberration chiefly depends. Therefore, in the construction of a compound lens for unit­ing the different colours, we may neglect this correction for the thickness and distance till the end of the process. When we apply it, we shall find that it chiefly affects the final focal distance, making it somewhat longer, but has hardly any influence either on the chromatic or spherical aberration. We do not hesitate to say, that the final formulæ here given are abundantly accurate, while they are vastly more ma­nageable than those employed by Euler or D’Alembert. We have calculated trigonometrically the progress of the rays through one of the glasses, which will be given as an example, giving it a very extravagant aperture, that the er­rors of the formulæ might be very remarkable. We found the real aberration exceed the aberration assigned by the formula by no more than 1/59th part, a difference which is quite insignificant. The process here given derives its sim­plicity from the frequent occurrence of harmonic proportions in all optical theorems. This enabled M. Clairaut to em­ploy the reciprocals of the radii and distances with so much simplicity and generality.

We consider it as another advantage of M. Clairaut’s me­thod, that it gives, by the way, formulæ for the more ordi­nary questions in optics, which are of wonderful simplicity, and most easily remembered. The chief problems in the elementary construction of optical instruments relate to the focal distances of central rays. This determines the focal distances and arrangement of the glasses. All the rest may be called the refinement of optics ; teaching us how to avoid or correct the indistinctness, the colours, and the distortions, which are produced in the images formed by these simple constructions.

Let *m* be to 1 as the sine of incidence to the sine of re­fraction ; let *a* and *b* be the radii of the anterior and pos­terior surfaces of a lens ; let *r* be the distance of the ra­diant point, or the focus of incident central rays, and *f* the distance of the conjugate focus ; and let *p* be the principal focal distance of the lens, or the focal distance of parallel rays. Make 1/*n* equal to 1/*a* — 1/*b* ; let the same letters *a*', *b', r,,* &c. express the same things for a second lens ; and *a", b'', r'',* &c. express them for a third ; and so on. Then we have 1/*f* = *—*1.1.1 *ml—*l .1 1 *m"—*1 1

*f~~~~n~ + ~r' f'~ ri " +r,', fi~ n" +√^',\*c∙* Therefore, when the incident light is parallel, and *r* infinite,

1 1 rn— l 1 m'—1 1 *m"-^l t*

we have - = ; -, = -l— ; — = *—, &c.*

*p n pl ri ρ\* n"*

And when several lenses are contiguous, so that their in­tervals may be neglected, and *therefore* 1/*f,* belonging to the first lens, becomes 1/*r*, belonging to the second, we have

1 \_ 1 - r, ~ 1 1 \_ 1 I.

*'r,~∕'~ n +r~p+ r,*

2.1 = ∙i = ⅛=-, + — +1 = 2 +1 +15

*rn J' ri n l r pl p r*

**a 1 rn"-1 1 m'—1 , m —1 1 1 1,1 1 1**

\*?=—+—+—+?=

Nothing can be more easily remembered than these for­mulæ, how numerous soever the glasses may be.

Having thus obtained the necessary analysis and formula, it now remains to apply them to the construction of achro­matic lenses, in which it fortunately happens that the em­ployment of several surfaces, in order to produce the union of the differently refrangible rays; enables us at the same time to employ them for correcting each other’s spherical aberration.

A white or compounded ray is separated by refraction into its component coloured rays, and they are diffused over a small angular space. Thus it appears, that the glass used