being also, in this case, = \*\*\*\*\*\*\*√(l — Tt) = 1 — and λ\_, , *AA j t\_* 8CC \_8B — 2ΛΛ , *t ~ + SCC’* aλ-8CC+ *AA~ SB — AA ~*

z∣

-j-^, corresponding to the verse sine of the time

*A*

or to the arc g^τ> in the circle represented by *Ct.*

*Corollary* 2. It follows that both *υ* and s must vanish continually at equal successive intervals, whenever ta. *Ct = 7∖*

***~~2 fΓ~~***> an(i n'hen cos. *ct* = 0, respectively ; the descent to the lowest point will therefore occupy the time correspond- τ ∠1 , , 1 *τ A .*

ιng to j -f- ≡g, and the subsequent ascent to - — ≡g : the

„ , , — U<

extent of the vibrations being always proportional to *e*

*Corollary* 3. The greatest velocity must take place at the point where *A* γ' -f- *Bs* = 0, and *AC* ta. *Ct* + ⅜√42 = *B,* or *t \_ B-∖AA . t r, AC .e*

.dC *I» — jfAA*

glect *A3,* cot. *Ct = -A* = cos. *Ct —* -, very nearly.

*t5 ζ*

*corollary* 4. The diminution of the successive vibra­tions is expressed by the multiplier *e* ^ai, which, when

*A A*

*ct =z* 2<r, the whole circumference, is 1 — — w, and yv *τλ,* O C√

*AerX*

or —— is the diminution of the value of *s* when the pen-

√zr

dulum returns to the place from which it first set out, that is, the difference between the lengths of two vibrations, each corresponding to a semicircumference, and this differ­ed *A*

ence is to —-s(, or —7= λ, the displacement of the point of *∖fi B ∖f L>*

greatest velocity, which measures the greatest resistance, as *π* to 1, or as 3·1416 to 1. We have seen that, for a re­sistance varying as the square of the velocity, this propor­tion was as 8 to 3, or as 2∙667 to 1.

*corollary* 5. If the pendulum be supposed to vibrate in a second, the unity of time, the diminution of the arc 2λ in each vibration will be 1/2*A* × 2λ, and the successive lengths will vary as \*\*\*\*\*e- 2λ, *e~A* 2λ, and so forth ; and after

the number *N* of vibrations, the extent of the arc will be reduced from 2λ to \*\*\*\*\**e -* ^ka 2λ; so that if we make \*\*\*\*\**e* — *Wa*

2 1

*= M,* we have hlJf = — *⅛NA,* and *A =* Thus,

2 if in an hour the vibrations were reduced to - of their ex­tent, which is rather more than appears to have happened in any of Captain Kater’s experiments, we should have *N* \_ 2 1

= 3600, and *M =* -, whence *A =* × ∙405465I

= ∙00022526, and J2 = ∙000 000 050 75; and since *B* = ^ = 9∙81, C=√(27-H2)= vW(l-^) = V-δ(l — *QB*) = V— ⅛⅛) ’ t'le faction being only ≡ ∙000 000 000 65 ; or about one second in 1600 millions, that is, in about fifty years.

*Scholium* 3. Although the isochronism of a pendulum, with a resistance proportional to the velocity, was demon­strated by Newton, yet Euler appears to have failed in his attempts to carry the theory of such vibrations to perfec­tion ; for he observes (*Mechan.* ii. p. 312), *Etsi ex his ap­pareat, tempora tam ascensuum quam descensuum inter se esse aqualia, tamen determinari non potest, quantum sit tempus sive descensuum sive ascensuum : neque etiam tem­pora descensuum et ascensuum inter se possunt comparari. Æquatio enim rationem inter s et u definiens ita est com­plicata, ut ex ea elementum temporis ds/*u*, per unicam vari­abilem non possit exprimi.*

*Scholium* 4. In confirmation of the solution that has been here proposed, it may not be superfluous to show the truth of the result in a different manner. Taking \*\*\*\*\*\* s= *ςemt* cos. *ct,* we have *= temt (rn* cos. *ct — c* sin. *ct),*

and *= icml (m3* cos. *ct — Cm* sin. *ct — cm* sin. *ct — c3* cos. *ct) ;* whence *+ A -∣- Bs = ςem, (m3* co3. *ct —* 2 *cm* sin. *ct — ci* cos. *ct -f- Am* cos. *Ct— Ac* sin. *ct -∣- B* cos. *ci) —* 0, and (m2— Cs -∣- *Am* q- *B)* cos. *ct—* (2Cm -f- *Ac)* sin. Ci = 0 : an equa­tion which is obviously true when the co-officients of both its terms vanish, and 2C*m* = — *Ac,* or *m* = ∣.4 ; and again *ci* = ms + *Am -∣- B = ∣ A3 — ∣ A\* -∣- B = B* — I *A3.* The former mode of investigation is more gene­ral, and more strictly analytical ; but this latter is of readier application in more complicated cases, and it will hereafter be further pursued.

*Lemma.* If a moveable body be actuated continually by a force equal to that which acts on a given pendulum, the body being in a state of rest when the pendulum is at the middle of its vibration, the space described in the time of a vibration will be to the length of the pendulum as the circumference of a circle is to its diameter. For the force being represented by cos. *Ct,* or cos. *x,* for the pendulum, it will become sin. *x* with regard to the beginning of the supposed motion, and the velocity, instead of sin. *x,* be­comes— cos. *x,* or 1 —cos. *x ;* so that the space, instead of 1 — cos. *x,* is *x —* sin. *x,* which, at the end of the se­mivibration, is *x =* instead of 1 — cos. *x* = 1, the space described by the simple pendulum, which is equal to its length.

*Scholium* 5. There is a paradox in the relations of the diminution of the vibration to the distance measuring the greatest resistance, which it will be worth while to consider, in order to guard ourselves against the too hasty adoption of some methods of approximation which appear at first sight unexceptionable. The pendulum, if it set out from a state of rest at the point of greatest resistance, would perform a vibration to the extent of double the distance of that point, or 2 λ, the initial force being measured by that distance. Now, when the resistance is very small, its magnitude may be obtained without sensible error from the velocity of the pendulum vibrating without resistance at the corresponding part of the arc ; and the velocity may be supposed to vary as sin. *Ct,* and the resistance, in the case of this proposition, as sin. *ct* or sin. *x* also. Hence it may be inferred by means of the Lemma, that the whole diminution of the space will be to *A/*√ *B* λ as *π* to 1, or that it will be equal to -*A/*√ *B* *π*λ, which has already