the whole from 2∙666*A*/√*Bλ*, the result of the more direct computation of Theorem G.

*Scholium.* An experimental illustration of the accuracy of the theorem may be found in the sympathetic vibrations of clocks, and in that of the inverted pendulum invented by Mr Hardy, as a test of the steadiness of a support (art. Pendulum, vol. xvii. p. 218) ; for since the extent of the regular periodical vibration is measured by *M/FF-B*, it isevident, that however small the quantity *M* may be, it will become very considerable when divided by *FF— B,* as *F* and √B approach to each other; and accordingly it is ob­served, that when the inverted pendulum is well adjusted to the rate of a clock, there is no pillar so steady as. not to communicate to it a very perceptible motion by its re­gular, though extremely minute, and otherwise impercep­tible change <>f place.

Theorem K. \*\*\*\*\* In order to determine the effect of a pe­riodical force, with a resistance proportional to the velocity, the equation —1--- -f- .4 -∣y -[∙ *lis — M* sin. *Gt* ≈ 0, may be satisfied by taking s = *a* sin. *Gt* -f- *β* cos. *Gt, a* being *= (GG — Hf + ÄÄGGM' and 3 =* (77σ=⅛y *ÄÄGG's being also = ÷ 8in∙*

(Ci + arcta.^) = ^GG~\_¾]iψ Jz1GG) si"' ∕λ, AG λ

arcta.B\_GGj.

Since *s= a* sin. *Gt -)∙ β* cos. *Gt, ~* = αGcos. *Gt — βG* sin. *Gt,* and -1⅛- =—<x(72sin.GZ—∕3G2cos. G⅛ =— G⅛,∙ di’

consequently the equation becomes (B—*G,) (a* sin. *Gt* -f- 3 cos. *Gt) + aA G* cos. *Gt—βAG* sin. *Gt-)■* Jf sin. *Gt* =0, and (B — *G3) a — βA G* -f- *M* = 0, and (B — *G1)β β AG*

*+ aAG* = 0 ; whence - = *-rr-τ-,* r>, also *S =*

α (j Cr *B*

*M—(GG — H)a aAG J)∖lf ∕r"*

*AG-j- = GG^Band ^-B)M-(G- lifa = aA^G-,* consequently *a =*

*A GM*

and *β = (GG-B****~~f + A~~~~3~~~~G^~~ ~~since,~~ ~~≡~~~~cnera1,~~ ~~'~~~~f~~***

*b =* ta. η, sin. *x -)∙ b* cos. *x* = √(l -j- 62) sin. *(x +* b) ; sin. *(x +* b) being =: sin. *x* cos. b -f- sin. B cos. *x —* cos. b (sin. *x* -p ta. β cos. *x),* and therefore sin. *x* -f- ta. b cos. *x =* ~~^ ws,t~~ ~~B)~~'-sin’ (z + b)8ec' B = sin.(≈ + B)√(l +^)t it follows that *a* sin. *Gt + β* cos. G7=α(sin. *Gt* -f- arc ta. -) √(l+g)1.∏a.√(l+S) = √(.∙ + i∙) =

*z (GG- By + AW M*

*([GG —By + A\*G')t* - √([GG-B]‘ + J«G‘)- *corollarg.* If we put *M* cos. *Gt* instead of *M* sin. *Gt,* we shall have « = <χ' sin. *Gt* -f- *β,* cos. *Gt ; a'* being = *β AGM*

*~ (GG — Bf + ÄW* nnt P - — “ -

GG ∕ ∕ GG \_ Bp -J- 4.4GG ant^ , ~ ^ + sιn∙

4- arc tβ. -j-)= √(β' + 3!) sin. (G7-∣-arc ta. —~~j θ~~~~g~~~~)∙~~

*Scholium* 1. Supposing *B* to approach very near to G2, a case very likely to occur in nature, because the effects which are produced, where it is found, will predominate over others, on account of the minuteness of the divisor; we may neglect the part of the denominator (*G2 — B)2,* in comparison with *A2G2,,* and the co-efficient determining\* will then become M/AG, the extent of the vibrations being inversely as *A* the co-efficient of the resistance ; and, in­deed, when the whole force of the periodical vibration is expended in overcoming a resistance proportional to the velocity, it may naturally be imagined that the velocity should be inversely as the resistance. It follows also from *G* the proposition, that in this case the arc ta. AG/B-GG approaching to a quadrant, the greatest excursions of the pe­riodical motion and of the free vibration will differ nearly one fourth of the time of a complete vibration from each other.

*Scholium* 2. Since *s* is a line, and *B* its numerical co­efficient, making it represent a force, and since sin. *Gt* is properly a number also, the co-efficient *M,* both here and in Theorem J, must be supposed to include another linear co-efficient, as *μ,* which converts the sine into a line, to be added to *s*, the distance from the middle point: that is, *M* must be considered as representing *Bμ,* in which *μ* is the true extent of the periodical change of the centre of sus­pension, and *B = 2g/l,* as in other cases ; so that *M* is

= 2g/lμ = 32 μ/l, and μ = Ml/2g = 1/32*Ml.*

*Corollarg.* In order to obtain a more general solution of the problem, we may combine the periodical motion thus determined with the free vibrations, as computed in Theorem H, the different motions, as well as the resist­ances, being totally independent of each other ; but the most interesting cases are those which are simply periodi­cal, the free vibration gradually diminishing with the mul­tiplier *e-mt,* and ultimately disappearing.

Theorem L. \*\*\*\*\*If there are several periodical forces, the equation ⅛ + *+ Βs + M*sin. *Gt* + JVsin. *Ft + ...*

≡ 0, may be satisfied by taking *s = a* sin. *Gt + β* cos. *Gt* 4- α' sin. *Ft* -j- *β,* cos. *Ft + ... =* ∕τr∙,⅛>——,⅛- ⅛

*τ ∖^Gi-Bγ+A∙G-)*

*. (r,, , AG ∖ 1* .V

*·'"■ (ff, - "c “■ β∑Tgg) +* + 7FP)

,⅛∙ (n-m K, +...

For, the equations expressing the space described being simply linear, the different motions and resistances are added or subtracted without any alteration of the respec­tive relations and effects.

*Scholium. A* free vibration may also be combined with this compound periodical vibration, by means of Theorem H ; but it will gradually disappear by the effect of the re­sistance.

*Lemma.* For the addition of the arcs *a* and *b,* begin­ning with the wcll-known equation sin. (α=+zZ∣) = sin. *a* cos. *b* ∑±= sin. *b* cos. *a,* we have, by addition, sin. *(a* -j- *b)* -∣- sin. *(a— b)-ii* sin. *α* cos. *b,* and sin. *a* cos. *b* = ⅜ sin. *(a* + i) -∣- ⅜ sin. (α — *b).* Then, if *c = b* + 90o, cos. ⅛ = sin. c, whence sin. a sin. c = I sin. (a -j- c — 90°) + ⅜ sin. *(a — c* + 90o) ; but sin. (i∙ + 90o) = cos. *x* and sin. *(x —* 90o) = — cos x, consequently sin. a sin. *c* = ∣ cos. (a — *c)—* ∣ cos. (a + c).