\*\*\*\*\* Again, if *e ≡ a —* 90o, cos. c = sin. α, and cos. *c* cos. *b = ⅞* sin. (α -∣- 6) -∣- ⅜ 8'n∙ (σ—⅛) = ⅜ sin. (c + 9θ° + *h)* + ⅛ sin. (c + f)O° — *l>) = i* cos. (c -f- *b)* -∣-1 cos. (c—*b).* Also, since cos. *α* cos. *b =* ⅜ cos. *(a* 4- *b)* 4-⅛ cos. *(a—b),* and sin. α sin. *b = ⅛* cos. (a—*b) —* A cos. (a 4- *b),* we have, by subtrac­tion, cos. (« 4- *b)* = cos. *a* cos. *b —* sin. *a* sin. *b,* and, by ad­dition, cos. (α — *b)* = cos. a cos. *b* 4- sin. *a* sin. *b.*

*corollary.* If *α* 4- *b — c* and *a — b = d,* cos. c 4- cos. *d c + d c—d λ j ο ■ c+fl*

— 2 cos. - — cos. —— ; and cos. α — cos. c = 2 sin. —— c — *d , . . , . a4-b α—b 1*

*cos.* —3— ; also sin. *a* 4- sm. *bzz2* sin. ' — cos. —— ; and

. , . , ∙ ∕ IV o ∙ α—b α + i

sin. *a —* sin.6 = sin. *a* 4- sin. (—*b) —* 2 sin. —— cos. —~.

THEOREM M. The equation, 4- *A* 'j\* 4- *Bs* 4- *Ii* sin. *Ft* sin. *Gt* ≡ 0, may be solved by taking s = α sin. ([F- G] *t* 4- *p) — β* sin. ([Z', 4- G] *t* 4- 7) ; α being

i"

√([(F- G)2-B]a 4- *At (F — G)r),*  1\* *β* = √([(F4- G)i--B]1 +W+ G,)s),

*B-(F-Gy 1 B-(F+cy*

*p=ατc*te∙-T(F∑=^'an^=arcta∙τ4(7-+-G) ∙

For since sin. *Ft* sin. *Gt = ⅛* cos. (27— *G) t—* ⅜ cos. *(F + G) t,* the equation becomes 4· *A* ⅛ 4- *Bs —* ⅜ Λcos, *(F -)- G)t* 4- ⅜ *H* cos.(F— *G)t = 0;* whence we obtain the solution by comparison with Theorem K and its corollary.

Sect. IV.—*Astronomical Determination of the Periodical Forces which Act on the Sea or on a Lake.*

In order to compute, by means of the theory which has been laid down in the two preceding sections, the primitive tides of any sea or any portion of the ocean, we must com­pare its spontaneous oscillations with those of a narrow pris­matic canal, situated in a given direction with respect to the meridian, which in general must be that of the greatest length of the sea in question, neglecting altogether the ac­tual breadth of the sea, which, if considerable, may require to have its own distinct vibrations compounded with those of the length, each being first computed independently of the other. Now, supposing the time required for the prin­cipal spontaneous oscillation of the sea or lake to be known, we must find the length of the synchronous pendulum, and taking *B = 2g/l = 32/l,* we must next find a series for ex­pressing the force in terms of the sine, or cosines of multi­ple arcs, increasing uniformly with the time.

Now the force is measured, for the direction of the meri­dian of the spheroid of equilibrium, by sin. cos. *z* (Theo­rem A), *z* being either the zenith distance or the altitude; and it is obvious that, when the canal is situated obliquely with respect to the meridian of the spheroid, the inclination of the surface, and with it the force, will be diminished as the secant of the obliquity increases, or as the cosine of the obliquity diminishes; so that the force will vary as sin. cos. *Alt.* sin. *Az.* if the canal be in an easterly and westerly di­rection ; or if it deviate from that direction in a given angle, as sin. cos. *Alt.* sin. *(Az.* + *Dev.);* and it is obvious that this force will vanish both when the luminary is in the ho­rizon, and when it is in the vertical circle, perpendicular to the direction of the canal ; that is, if we consider the force as acting horizontally on a particle at the middle of the length of the given canal ; and the same force may be con­sidered as acting vertically, with a proper reduction of its magnitude, at the end of the canal ; for tne horizontal oscil­lations at the middle must obviously follow the same laws as the vertical motions at the end.

The case, however, of a canal running east and west, ad­mits a very remarkable simplification ; and since it approaches nearly to that of an open ocean, which has been most com­monly considered, it will be amply sufficient for the illus­tration of the present theory. For, in general, sin. *Az.*

\*\*\*\*\*\*\*cos. *Deel.* sin. *Hor. <c . . . . ...*

— 77 -, and the expression, sm. cos. *Alt.*

cos. *AU. r*

sin. *Az.,* becomes in this case sin. *Alt.* cos. *Decl.* sin. *llor.* ∙≤. But sin. *Alt.* ≡≡ sin. *(Lat.)* sin. *Decl.* + cos. *(Lat.)* cos. *Decl.* cos. *IIor.* ∙≤, and calling sin. *(Lat.)* for the given canal L, and cos. *(Lot.)* l,, the force becomes n sin. cos. *Decl.* sin. *Hor.* <[ 4- L, cos.2 *Decl.* sin. cos. *Ηor.* . Now, sin. *Decl. =* cos. *Obl. Bel.* sin. *Lat.* 4- sin. *Obl. Bcl.* cos.

*Lat.* sin. *Long. ;* and since cos. *φ —* 1 — ∣ sin.2 P 4^ ïj s\*u∙\* 5

*φ—* jjjsin.β *φ* 4- ..., the true value of cos. *Decl.* might be expressed, if required, by means of this series, and its second and fourth powers would in general be sufficient for the computation.

But it will be more convenient to suppose the sun and moon to move in the ecliptic, and the ecliptic to be at the same time so little inclined to the equator, that the longi­tude may be substituted for the right ascension ; a substi­tution which will cause but little alteration in the common phenomena of the tides. Then, if the sun’s longitude be Θ, and the moon’s J , the horary angles *t* and *t',* and the sine of the obliquity of the ecliptic *ce,* we shall have sin. *Dccl. — α* sin. ©, or *ce* sin. J ; and cos. *Deeb* =1 — sin.2

3 3

*Decl.* 4- - sin.4 *Dec!.* = 1 — ⅜ cbs sin.’ © 4- - *<ι'* sin.4 Θ ..., o α

and sin. cos. *Decl.* = œ sin. Θ — ] *ce3* sin.’ © 4- ∣ <ei sin.5 © ... ; also cos.' *Decl. —* 1 —*(t°* sin.' Θ ; whence the

3 sun’s force becomes 1. sin. *t (<e* sin. Θ — ⅜ *α3* sin.3 © 4- -

Ö cb5 sin.5 © ...) 4- I? ⅜ sin. *2t* (1 — ce2 sin.s Θ) = 1. sin. *t* (œ sin. © — ⅛ α\*5 (5 sin. ® — 7 s\*n∙ 5 ®) + ζ ar' s'ln∙ © — ∙∩τ sin. 3 © + sin. 5 ©4- l’ sin.2∕(l — lJα2 4- ⅛ α≡2 cos. 2 ©) “ L sin. *t* (of sin. © 4- *<e"* sin. 3 © 4^ *ee','* sin. 5Θ)-f-(^L — I l' œ2) sin. *2t* 4- ] l' <e- sin. *2t* cos.

3 15

2© ; œ' being *= œ — — οe3* 4- — *a?* ... ; or about ∙3645 ; o 64

1 15 3

*a? = - a? —* œ5 ... = ∙0078, and *af' =* <r5 ... —

8 128 128

•00002, and cεs = 4585. But sin. *t* sin. © = J cos. *(t — ©)*

— ⅜ cos. *(t* 4- ©), and sin. 2 *t* cos. 2 © = A sin. 2 (t 4- ©) 4- 1[ sin. 2 *(t — © ).* Hence the sun’s force becomes S’ (l <e, [∣ cos. *(t —* ©) — I cos. *(t* 4- ©)] -∣- L *af* [⅛ cos. *(t —* 3 ©) — I cos. *(t* 4- 3 ©)] 4- L βf [∣ cos. *(t —* 5 ©)

— ⅛cos. (t 4- 5©)] 4- ⅛(l-⅛α2) sin. 2/ 4- \*^α"[Jsin.2 («4- ©) 4- I ein. 2 *(r—* ©)]) ; and that of the moon may be expressed in the same manner, by substituting *M, t',* and

J , for <S, *t,* and ©.