The effect of that part of the hydraulic resistance, which is proportional to the square of the velocity, must be ex­pressed by an approximation deduced from the periodical character of the force, as depending on that of the primitive forces concerned ; taking, however, the precaution to use such expressions only as will always represent this resist­ance in its proper character as a retarding force : for if we simply found for it an equivalent expression, denoting ac­curately the square of the velocity, this square, being always positive, would imply a force acting always in the same di­rection. Now, we have already seen (Theorem J, Cor. 3), that sin.2 *x* may be considered, with respect to its principal effect, as equivalent to ∙8484 sin. *x :* and if we neglect, in the determination of the resistance, the effect of the smaller forces, and compute only that of the principal terms 1/2 L'sin. *2t,* and 1/2 l' sin. *2t',* we may call the velocities depend­ing on these forces 6’ cos. (2*t* + *s*’) and *M'* cos. (2*t'* + *m'); S*' and *M'* representing not exactly the proportion of the primitive forces of the sun and moon, but that of the tides depending on their combination with the conditions of the given sea or lake. The resistance will then be as the square of \*\*\* *S'* [cos. (2*t* -∣- s') 4- cos. (2∕' -f- nι')] -f- *(M'— S)* cos. (2t' 4- *m')* ; and when least, it will be *I) (M'— S')“,* and when greatest, *D(M' -∣- S,)1,* the difference being *∖∙DM'S∙,* so that the difference may be sufficiently represented by *Μ)M'S* [cos. (2t -∣- s') 4- cos. (2t' 4- wι')] × ∙8484, or ra­ther × (∙8484)i, because the value of cos. *t* 4- cos. <, = 2 cos. cos∙ which is to be squared, requires thc

reduction from 1 to ∙8484 for each of its factors ; and in this manner we obtain a perfect representation of the period and quality of the resistance, and a very near approximation to its magnitude.

lt will, however, be still more accurate to consider the resistance thus determined as comprehended in the value of the co-efficient *A,* substituting for it, in the case of the solar tide, *A'* = *A* + 2∙88 *DM',* and for the moon *A"* = *A* + 2∙88 *DS'* 4- ∙8484 *D (M'— S)∙,* this latter part ex­pressing that portion of the resistance *D* which observes the period of the lunar tide, and which may therefore be con­sidered as added to the resistance *A* for that tide only.

\*\*\*\*\*\*Hence, collecting all the forces concerned into a single equation, the expression will become 4- √4w⅛4^^i + <S, (1. *cd* [∣ cos. (< — © ) — ) cos. (i 4- Θ ) 4- L *ce”* [∣ cos. *(t—* 3 ©) — J cos. *(r* 4- 3 ©)] 4- *cd"* [.J cos. *(t —* 5 Θ) — cos. (<4-^ Θ)] + (1 — <®\*) sin. 2t 4- j αs [∣ sin.

2(t4-Θ)4-⅜ sin. 2 *(I —* Θ ) ]) 4- *M (l ed* [ J cos. *(l'-*5 ) —⅛cos.(t' 4. J) )] 4- L<r"[⅛co3√<-3 J ) —⅜ cos. *(f* 4- 3 J )] 4- L *cd"* [Î cos. *(t' — 5* J ) — I cos. (t, 4- 5 j )] 4- g (1 — *as)* sin. 2z' 4- œ2 [⅜ sin. 2 (f-∣- J ) 4- I sin. 2 *(d —* J )]) = 0 ; and from each of these terms the value of the corresponding pair of terms in the value of *s* may be obtained independently, by comparison with the *M* sin. *Gt* or *N* cos. *Gt of* Theorem K, which gives us *(GG — B)* sin. *Gr* 4- *A G* cos. *Gt ιr .*

*(GG~Bγ + AAGG J/’ and AG* sin. *Gt* 4- *(B— GG)* cos. *Gt „ . 1*

*(GG- Bf + AAGG* 3, resPect,vely∙

But without entering minutely into the effects of all the terms of the equation of the forces, it mav be observed in general that their results, with regard to the space describ­

ed, will not differ much from the proportion of the forces, except when their periods approach nearly to that of the spontaneous oscillation, represented by *B.* Thus, since ⅛ cos. *(t —* Θ) — ⅜ cos. *(t* 4- ©) is the representative of sin. *t* sin. Θ, and since these terms will afford results in the form I α cos. *(t—* 0) 4^ ⅜β s>n∙ (t— ©), an^ 0\* ⅛ cos∙ (∕ 4- *Q)* 4. £ *β'* sin. *(t* -∣- Θ), and if we neglected the slight difference of *a* and *a',* which is that of 1 — y-) — *B,* and (1 4- ® ) — 2?, y∙ being ~~36~~~~⅛-2⅛4~~ ~~οη1~~~~?’~~ ~~we should~~ have ⅛α [cos. *(t — a) —* cos. *(t* 4. ©)] 4- [s\*n∙ (t— ®)

— sin. *(t* -∣- 0 ) = α sin. *t* sin. 0 4- *p* cos. *t* sin. 0 = sin. θ (a sin. *t + ß* cos. /)]; which is the same as if we consi­dered the effect of the force sin. *t* separately, and afterwards reduced it in the proportion of sin. 0. Hence it is obvi­ous, that for all modifications of the forces greatly exceeding in their periods the period of spontaneous oscillation, the effects may be computed as if the forces were exempt from those modifications, and then supposed to be varied in the same proportion as the forces ; but we cannot be quite cer­tain of the magnitude of the error thus introduced, unless we know the exact value of *B,* which determines the time of spontaneous oscillation.

\*\*\*Considering, therefore, in this simple point of view, the correct expression of the force l sin. cos. *Decl.* sin. *Hor.* ∙≤ 4- l' cos.2 *Deel.* sin. cos. *Hor.* ≤, or sin. 2 *Decl.* sin. *Hοr.* < 4- Jι.' cos.’ *Decl.* sin. 2 *Hor.* ≤, we may ob­serve that the phenomena for each luminary will be ar­ranged in two principal divisions, the most considerable being represented by ⅛l' cos.2 *Decl.* sin. 2 *Hor.* <≤, and giving a tide every twelve hours, which varies in magni­tude as the square of the cosine of the declination varies, increasing and diminishing twice a year, being also propor­tional to the cosine of the latitude of the place, and disap­pearing for a sea situated at the pole. The second part is a diurnal tide, proportional to the sine of the latitude of the given canal, being greatest when the luminary is far­thest from the equinox, and vanishing when its declination vanishes.

From these general principles, an attentive student may easily trace for himself the agreement of the theory here explained, with the various modifications of the tides as they are actually observed. It however remains for us to inquire more particularly into the cause of the hitherto un­intelligible fact, that the maximum of the spring tides in the most exposed situations is at least half a day, if not a whole day, later than the maximum of the moving forces.

Now it is easy to perceive, that since the resistance ob­serving the lunar period is more considerable than that which affects the solar tide, the lunar tide will be more re­tarded or accelerated than the solar ; retarded when the oscillation is direct, or when *G2 — B* is positive, and acce­lerated when it is inverted, or when that quantity is nega­tive ; and that in order to obtain the perfect coincidence of the respective high waters, the moon must be farther from the meridian of the place than the sun ; so that the great­est direct tides ought to happen a little before the syzygies, and the greatest inverted tides a little after ; and from this consideration, as well as from some others, it seems pro­bable that the primitive tides which affect most of our har­bours are rather inverted than direct.

If we wish to apply this theory with precision to the actual state of the solar and lunar motions, we must deter­mine the value of the co-efficients, from the tables of those luminaries. And, first, making thc unit of time a whole solar day, in which the horary angle *t* extends from 0° to 360°, the sun’s mean longitude will be *t*/365·254, added to the longitude at the given epoch, and the moon’s approxi-