Benlomond A = 56° 43' 28''·58

Cairnsmuir. B — 79 42 28 ∙69

Knocklayd C = 43 34 36 ∙89

Sum 180 0 34 16

together with the chord of the arc joining A and B, obtained from the computation of a former triangle, viz. chord c = 352,033·48 feet.

The first step of the operation is to compute approximate values of the two sides *a, b.* For this purpose we have the formulæ \*\*\**a = , b =.* ~~C s~~~~^~~~~n~~~~'J\*~~ ; and it will be sufficient

sm. C sin. C

to use logarithms to five places.

\*\*\*Log. *e ~* 5∙54658 Log. *c —* 5∙54658

Log. sin. A = 9∙92223 Log. sin. B = 9 99295

Co. log. sin. C ≡ 0Ί6158 Co. log. sin. C =0’16158

Log. *α =* 5∙63039 Log. \* = 5’70111

Hence the approximate values of the three sides are *a* = 426,960, *b =* 502,470, c = 352,030 feet.

We have next to find the corrections for reducing the observed angles to the chord angles, and for this purpose it is necessary to have an approximate value of the curvature of the surface. Now the approximate latitude of Benlomond (the northernmost station) is 56° 11', and that of Knocklayd (the southernmost) is 55° 10'; the mean of the two is 55° 40' ; and if we adopt this as the value of *l,* and assume the elements of the spheroid given in the article Figure of the Earth, p. 563, viz. *a* =*.* half the polar axis = 20,852,394 feet, \*\*\* *e = -—— =* ~~u~~~~,,,∖gg~~ = ’00:1322, *a* 301∙02o

the formulæ (2), (3), and (4), give the following values of R, R', and *r* (the radii of curvature in the direction of the meridian, the direction perpendicular to the meridian, and the direction of a line making an angle of 45° with the me­ridian) in feet, viz.

R = 20,924,824, R' = 20,968,900, *r =* 20,946,814.

With the value of *r* thus found, we proceed to find the chord angles corresponding to the observed angles A, B, C, by means of equation (6). And first to find *x,* the correc­tion for A.

\*\*\*\*Log. (A + c) = 5∙93l7l Log. (5 — *c)* =547734 Log. r = 7∙321l2 Log. *r =* 7-82112

8∙6l059 7∙85622

2 2

7∙221l8 5-71244

Log. tan. I A = 9∙73235 Log. cot. ⅜ A = 0∙26765 Co. log. 16 = 8∙79588 Co. log. 16 = 8-79588 Log. R" = 5∙31443 Log. R" = 5∙31443

11'∙583 = l∙06384 l'∙231 = 0∙09040 Hence *x* = 11"∙583 —l"∙23l = 10,∙352. As the cor­rections for the other angles B and C are found pre­cisely in the same manner, it is unnecessary to give the calculation. The results are respectively *x, =* l4"∙684, *x*" = 9''∙724.

From this we have the spherical excess E (= *x* + *x' + x'')* = 34"∙760. But the excess of the sum of the three ob­served angles above 180° was 34"·16, consequently the error of the observations is — 0"·60. Now, on dividing this error into three parts proportionally to the three numbers ∙96l, 10, 2·146 (the reciprocals of the weights), we find the cor­rections for the observed angles to be respectively + 0''·04, + 0"·46, and + 0''·10. On applying these corrections, we obtain the true spherical angles ; and on diminishing each of these last by the corresponding corrections for reduction to the chords, viz. 10''∙35, 14"∙68, 9"∙73, we obtain the chord angles, or angles for calculation. The results are as fol­lows :

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Observed Angles.** | **Spherical Angles.** | **Chord Angles.** |
| A  B  C | 56° 43' 28"∙58  79 42 28 ∙69  43 34 36 ∙89 | 56° 43' 28' ∙62  79 42 29 ·15  43 44 36 ∙99 | 56° 43'18''∙27  79 42 14 ·47  43 34 27∙26 |
|  | 180 0 34 ·16 | 180 0 34 ∙76 | 180 0 0∙00 |

With the angles in the last column of this table, we are enabled to calculate the lengths of the chords opposite A and B. It is now necessary to use logarithms to seven de­cimals at least. The calculation is as follows.

\*\*\*Log. chord c (= 352033’48) = 5∙5465840 Log. sin. A (56® 43' I8"∙27) = 9∙9222l44 Log. sin. B (79 42 14 ∙47) = 9∙9929499 Co. log. sin. C (43 34 27 ∙26) = 04615956

Log. chord *a —* 5∙6303940

Log. chord δ =5∙70ll295

and the results are, chord *a* = 426,966∙69 feet, chord *b* = 502,492∙46 feet, chord c = 352,033\*48 feet.

It now only remains to determine the spherical arcs corresponding to the above chords. Using the same va­lue of the radius as above, namely, log. *r* = 7∙32112, we find

\*\*\*\*⅛=∙oo>⅛ = iSi = \*∙n∙

and therefore, by equation (7), the distances on the surface of the spheroid are,

Cairnsmuir to Knocklayd *a =* 426,974∙08

Benlomond to Cairnsmuir *b =* 502,504∙51

Benlomond to Knocklayd c = 352,037∙62

The whole process is brought under one view, by ar­ranging the calculations as in the subjoined table.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Stations** | **Observed Angles.** | **Observations.** | | **Chord Cor­rections.** | **Appor. of Error.** | **Angles for Calculation.** | **Calculation.** | **Chord Sides opposite each**  **Angle.** | **Lengths of Arcs.** |
| **No.** | **Recip. of**  **Weight** |
| Benlomond... | 56°43'28''·58 | 3 | 0∙961 | — 10∙35 | + 0∙04 | 56° 43' 18"∙27 | 5∙5465840  9∙9222144 | 426966·69 | 426974∙08 |
| Cairnsmuir... | 79 42 28 ∙69 | 1 | 10Ό00 | — 14∙68 | + 0∙46 | 79 42 14 ∙47 | 9∙9929499 | 502492·46 | 502504·51 |
| Knocklayd.... | 43 34 36 ∙89 | 2 | 2Ί46 | — 9∙73 | + 0∙10 | 43 34 27 ∙26 | 0∙1615956 | 352033∙48 | 352037∙62 |
|  | 180 0 34 ·16 Error, — 0 ∙60 |  |  | 34∙76 |  |  | 5∙6303940  5∙7011295 |  |  |