likewise for the computation of the spherical excess, are given by Delambre in the first volume of the *Base Métrique,* and also in Puissant’s *Traité de Géodésie.*

When two angles only of a triangle were observed, the practice adopted in the Ordnance survey was to compute the reduction of the two observed angles to the plane of the chords, and to give the angles thus reduced, as the angles for calculation, true to the nearest quarter of a second. The third angle was then assumed, so that when added to the two chord angles the sum was 180°.

*calculation of the Latitudes, Longitudes, and Azimuths.*

The formulæ which have now been given suffice for the computation of the sides of the principal triangles, or the distances of the stations from each other ; but for the pur­pose of mapping the country, it is also necessary to deter­mine the latitudes and longitudes of the several stations, and the inclinations of the sides of the triangles to the me­ridian. In order to effect this by means of the geodetical measurements, the latitude of one station at least, and the inclination of a side of one of the triangles to the meridian of that station, must be determined by astronomical means ; and when this has been done, the geographical latitudes and longitudes of all the other stations in a chain of tri­angles, and the inclinations of their several sides to the meridians which pass through their extremities, may be computed. The computation, however, is of such a nature that a small error in determining the direction of the meri­dian leads to errors of considerable magnitude in the lati­tudes and longitudes deduced from it ; and it accordingly becomes necessary, in the progress of a large survey, to verify the computed results by astronomical observations at various stations not very remote from each other in re­spect of longitude. It has been stated in the preceding historical sketch, that eight different meridians were deter­mined by direct observation in the progress of the survey of England. The problem which usually occurs in prac­tice may be enunciated as follows.

Given the latitude and longitude of a station A, and the distance of A from another station B, and also the direction of the meridian at A in respect of the straight line AB, to determine the latitude and longitude of B, and the direc­tion of the meridian at B in respect of AB.

On the surface of a sphere whose centre is at the point where the normal at A intersects the polar axis of the spheroid, let PAX be the meridian of A, PBY the me­ridian of B, meeting the former in P ; then in the spherical triangle PAB there are given PA the co­latitude of A, the side AB deter­mined from the triangulation, and the observed angle PAB which (in the annexed figure) is the sup­plement of the azimuth at A, the azimuths being supposed to be reckoned from the south towards the west, all round the circle; and from these data we have to deduce PB the co-latitude of B, the angle APB the difference of the lon­gitudes of A and B, and PB A, which, added to 180°, is the azimuth of A as seen from B.

\*\*\*Let *l* = 90o — AP = latitude of A, *l'* = 90° — BP — latitude of B, λ *=l — t,*

A = 180° — PAB = azimuth of B seen from A, *d* = arc AB in parts of the radius.

In the spherical triangle PAB we have

cos. PB = sin. PA sin. AB cos. PAB -∣- cos. PA cos. AB ; that is (since cos. P AB =—cos. ( 180o — PA B) = — cos. A),

sin. *t* ≡ — cos. *l* sin. *d* cos. A -f- sin. *l* cos. *d,* whence

sin. *I—* sin. *t* = cos. *I* sin. *d* cos. A -p sin. *l(l —* cos. *d).* But sin. *l —* sin. *f = 2* sin. *£ (l — l')* cos. ⅛ (Z + O = 2 sin. ⅛ λ cos. (Z — I λ)

= 2 sin. ∣ λ (cos. Z cos. J λ -f- siu. Z sin. ⅛ λ), therefore

2 sin. I λ (cos. Z cos. ⅜ λ -p sin. Z sin. ∣ λ) = cos. *l* sin. *d* cos. ? -∣- sin. Z (1 — cos. d).

\*\*\*Now since λ the difference of the latitudes can only be ι small arc, we may reject without sensible error all th< terms in the development of the sine and cosine contain ing higher powers of ∣ λ than the square ; that is, w< may assume 2 sin. ∣ λ = λ, cos. J λ = 1 — ⅜ (i λ)a sin. I λ = ⅜ λ. In like manner, since *d* is small in comparisor of the radius, we may put sin. *d = d,* 1 — cos. *d = ⅛ d! ;* whence the above equation becomes, on dividing by cos. *l, λz= d cos.* A -∣- J *(d1 —* λs) tan. Z. (9).

For a first approximation the second term may be ne­glected, and we have then λ = *d* cos. A, which is equiva­lent to supposing the meridians at the two stations to be parallel. Substituting this value of λ in the second term, we get for a nearer approximation

λ = *d* cos. A -∣- A *d'i* sin.’ A tan. Z (JO).

This new value of λ may be again substituted in the equa­tion (9), by which a still nearer value would be obtained ; but it is unnecessary to carry the approximation farther.

The difference of latitudes λ is here expressed in parts of the radius 1. Let λ" be the number of seconds in λ; then λ = λ\* sin. 1’ = *(l — l')* sin. 1’ (Z — *l'* being also expressed in seconds). The arc *d* likewise denotes the dis­tance on the sphere from A to B in parts of the radius ; therefore, if D be the number of feet in AB, and R' the number of feet in the radius of the circle perpendicular to the meridian at A, we shall have *d* ≡ D ÷ R'. Making these substitutions, the last equation becomes

l j, D cos. A D2 sin.\* A tan. Z i,11 λ

l~c- R'sin. 1" 2 R'a sin. 1’ t '-

This formula gives the latitude of the station B, suppos­ing the surface that of a sphere ; but a small correction is required on account of the compression. To investigate this, let AM and BN be the normals to the spheroidal surface meeting the polar axis in M and N ; join BM, and draw MX and NY parallel to the plane of the equator. The angle AMX is the astronomical latitude of the station A, or the latitude Z, and the angle BMX is the latitude of B, determined by the above formula. But the true astronomical latitude of B is the angle BNY, which is less than BMX by the angle MBN ; we have therefore to compute this angle, and add its value to λ, the difference between Z and *t.* \*\*\*Let MBN = ∙ψ. In the triangle BMN we have sin. -ψ : sin. BNM : : MN : MB. Now sin. BNM = cos. BNY = cos. *I* ; and if C be the centre of the spheroid, then MN = CM — CN ; and be­cause AB is always a small arc on the spheroid, MB is sen­sibly equal to AM; therefore the above proportion gives AM sin. ∙ψ = (CM — CN) cos. *l'.* lf we now assume α and *b* to denote respectively the polar and equatorial semi­diameters, we readily find, from the equation of the ellipse, CM = ***~~b~~*** AM sin. Z. Substituting α (1 -∣-e) for 6, developing, and rejecting terms multiplied by powers of e, we have (δ2 — α\*) -⅛- *l>‘* = 2e; whence CM = 2e AM sin. Z. In like manner, CN = 2e AM sin. Z, ; consequently