determined, and also the amount of atmospheric refraction, which is an element of the same problem.

Let A and B be two stations whose distance from each other is known, and let AC and BC be the directions of gra­vity, meeting in C, the earth being supposed spherical. Let AD and BD be the horizontal lines in the plane ACB, and suppose *a* and *b* to be the ap­parent planes of A and B re­spectively, as seen from each other. Now, if the rays of light proceeded in straight lines, the angle DAB would be the depression of B below the horizon of A, and DBA the depression of A below the horizon of B ; and since AD and BD are perpendicular to AC and BC respectively, the sum of these two angles is equal to C, or to the arc of the great circle intercepted between A and B. But this arc is known from the measured distance of A from B, and the assumed dimensions of the earth ; consequently the sum of DAB and DBA is given. If, therefore, from this given sum we subtract the sum of the two observed de­pressions, namely, DA*b* and DB*a*, there will remain the sum of BA*b* and AB*a*, that is, the sum of the two refractions, the half of which is the mean refraction, and assumed to be the actual refraction at each station. Let us put

*d —* DA*b*, the observed depression at A, *d'* = DB*a*, the observed depression at B, C — ACB, the measure of the contained arc, *g* = the mean refraction, and we have

*g* = ½{C-(*d+d''*)} (15).

Now, let II be the point in CB which is on tl>e same level surface with A, so that CH = CA ; then HB is the altitude of B above A, which we have to determine. Join AH, and the angle DAH is equal to half of C. But the angle BAH is obviously the difference between DAH and the sum of DA*b* and *b*AB. Hence if *φ* denote the angle BAH, we have

*φ* = ½C-(*d+g*) (16).

This gives the angular elevation of B above A ; conse­quently if *φ* is expressed in seconds, and AB in feet, then BH will be obtained in feet from the formula

BH = AB× *φ*sin. 1’ (17).

It has been assumed that the objects observed at both stations are depressed. If one of them is elevated, then *d or* *d'* must be taken negatively. It is also to be remarked, that for the purpose of finding the refraction, each obser­vation must be reduced, previously to the calculation, to the axis of the instrument ; that is to say, allowance must be made for the difference of level between the axis of the instrument at either station, and the object (as the top of a flag-staff) at the same station observed from the other. This allowance is made by computing the angle subtended by that difference at the distance between the stations, and deducting it from the observed depression, or adding it to the observed elevation.

For the determination of the absolute altitudes, it is ne­cessary that the heights of one or more of the stations be ascertained by actually levelling down to the surface of the sea. the heights of all the intermediate stations are then established in the manner now explained, by the reciprocal angles of elevation or depression, carried on from station to station ; and it is obvious that a verification will be obtain­ed for every three stations ; for the difference of altitude between A and B, when found by direct observation, ought to be the same as when deduced from the difference of the heights of each of those stations, and a third station C.

For estimating the altitudes of places at which the theo­dolite was not placed, and where, consequently, reciprocal observations were not obtained, it was necessary to assume a mean value of the refraction *g,* in order to obtain the an­gle *φ* in the equation (16). On account of the wide limits within which the horizontal refraction is found to vary, this is attended with some uncertainty, even when every atten­tion is paid to the condition of the atmosphere. In gene­ral the effect of refraction was assumed to be from 1/12th to 1/14th of the whole difference of altitude.

As an example, we select the following case from the Survey. At the station of Black Comb in Cumberland, Scilly Bank appeared depressed 49' 14" ; and at Scilly Bank, Black Comb was observed elevated 31' 31”; and the distance between the two stations is 121,028 feet. From these data we have to compute the mean refraction and the difference of altitude.

The height of the instrument at both stations was 5½ feet. Hence if R' denote the number of seconds in the are equal to the radius, and *x* the correction for the height of the instrument, we have this proportion, 121028 : 5 ∙ 5 :: R'' : *x* ; whence, since R" = 206,264∙8, *x* is found = 9'. Sub­tracting this from the seconds of observed depression, and adding it to the seconds of observed elevation, we find *d* = 2944'' and — d’ = 1900" ; whence *d* + *d'* = 104 +''. Now allowing the length of a second of arc at the mean latitude to be 101½ feet, the distance in seconds is 121,028 ÷ 101½ = 1192, or C = 1192'', and we have by formula (15) the mean refraction *g* = ½ { C — *(d* + *d')}* = 74", which is very nearly 1/16th of the intercepted arc.

With this value of *g* we obtain (16) *φ* = — 2422''; whence, since sin. 1" = ∙00000485, we have for the differ­ence of altitudes (17)

121028 × —2422 × ∙00000485 = — 1422, or the depression of Scilly Bank below Black Comb is 1422 feet. In the Survey, vol. iii. the altitudes of the two sta­tions are stated to be respectively, Black Comb 1919 feet ; Scilly Bank 500 feet ; whence the difference is 1419 feet.

We have now explained, with as much detail as the limits which must be prescribed to the present article would per­mit, the methods of calculation by which the mutual dis­tances of the several stations of an extensive survey, and their geographical co-ordinates, that is to say, their lati­tudes, longitudes, and altitudes, are deduced from the ob­servations. These form the principal and essential objects of a trigonometrical survey of a country. There are how­ever various other points of great importance connected with geodetical measurements, into which, in the present outline, it is impossible for us to enter ; and particularly the methods of estimating the probable errors of the obser­vations, and of the results deduced from them, and of ap­plying these results to the determination of the form of the earth’s surface in the country over which the operation *ex­tends.* For information on these points, and indeed on all other questions relating to the higher geodesy, we refer the reader to the following works : Delambre, *Base du Systeme Métrique Decimal,* Paris, 1806-10 ; Gauss, *Supplementum Theories combinationis Observationum Erroribus minimis obnoxiœ,* Göttingen, 1828; Struve, *Breitengradmessung in den Ostsetproι∙iuzen Russlands,* Dorpat, 1831; Bessel, *Grad­messung in Ostpreussen und ihre Verbindung mit Preussichen und Russiehen Dreieehshetten,* Berlin*,* 1838; Puissant, *Traité de Géodésie,* 2d ed. Paris, 1822 ; and particularly the *Nou­velle Description Géométrique de la France,* Paris, 1832-40, by the last-named author,—a work which we could wish to see imitated in our own country. (s.)