tions are only approximations, they will diminish in accu­racy as we advance. We may however proceed otherwise thus. Find, as has been explained, a series of sines of arcs differing by 45' from 0° to 60° ; then fill up, by interpolation, the intervals thus: We have found (Algebra, art. 239) \*\*\*\*\*\*\*\*\*sin. *(a + l>)* + sin. (α — ⅛) = 2 .cos. *b* sin. a : instead of *a* write *x + h,* and instead of 6 write *h,* and we have sin. *lx + 2∕ι)* + sin. *x = 2* cos. *h* sin. (\* + Λ).

, , As , *P hβ*

Now cos. Ä = 1 — τ-5 + — — -.-~~j-.∙. γ - g»~~ ~~&c~~~~·~~

**Γ2 l∙2∙3∙4 l∙2∙o∙4∙5∙o**

*h\* h6*

Putn to denote the series *h- —∙ - + - , —-, —* &c. and U\*T m‘4'0’0

we have

sin. (.1· + 2A) + sin. *x* = 2 sin. (.r + *h) — n* sin. (x + A) ; and sin. *lx* + 2Λ) = sin. (r + A) + (sin. (r + A) —sin.a:) *— n* sin. *lx* ⅛ A). If A be an arc of one minute, then A = ∙0002908882, and *n* — ∙t)000000846. Let *x* denote any angle, and suppose A to be one minute; the formula gives

sin. (a: + l') = sin. *x* + (sin. *x* — sin. *lx —* Γ))

*— n* sin. *X,*

sin. *lx* + 2') = sin. *lx* + Γ) + (sin. *lx* + l') — sin.

*— n* sin. (a· + l').

sin. *lx* + 3') =sin. (® + 2') + (sin.*lx+i2')—*sin.(≈+ Γ))

*— n* sin. *lx* + 2').

Thus, from the sines of two arcs *x—* 1' and *x*, which differ by 1', and the constant number *n* = 2 ( 1 —cos. l') = (2 sin. 30'')2, the sines of all angles or arcs exceeding *x,* and differing by l', throughout the table, may be found. The most ex­tensive table of sines in existence (which is yet in manu­script), was computed under the direction of the late Μ. Prony in this way. It gives the sines of all arcs in a series of which the common difference is one ten-thousandth part of a quadrant with twenty-five decimals.

11. The sines being found, the tangents and secants may be obtained, each by a single division from the formulæ

tan.*a* = sin.*a*/cos.*a*, sec.*a* = 1/cos.*a*.

The tangents of the latter half of the quadrant may how­ever be found by addition only from the formula tan. (45° + *a)* = tan. (45° — *a*) + 2 tan. 2rr, which is a deduction from (4) of (O) Algebra, art. 248; and the secants from this formula,

sec. *a* = tan. *a* 4- tan. (45° — ½*a*), Algebra, 249.

12. With a table of sines, and tangents, and secants, the ordinary questions relating to triangles may be resolved by multiplication and division, but more expeditiously by lo­garithmic calculation ; and therefore, in general, mathema­tical tables give only the logarithmic sines, tangents, and sometimes secants, which, however, may be readily found from the log. cosines.

Part I.—Plane Trigonometry.

13. In contemplating a plane triangle as a figure formed by three straight lines, we distinguish the sides of the tri­angle, the angles at their intersections, and the space or area which they comprehend. In trigonometry we abstract from the area, and consider only the sides and angles. There are in all six angles, viz. the three interior and their adja­cent exterior angles. Each of the exterior is the sum of two interior angles, and of these any two determine the third ; so that in fact the independent elements of a triangle are five, viz. its three sides, and any two of its three interior angles; and of these any three being given, the remaining two may be found.

14. The sides and angles are dissimilar in their nature,

and therefore do not admit of being directly compared : the measures of angles, being arcs of a circle described with a given radius, are lines, and therefore expressible by the sides. It is however more convenient and easy to use the sines and tangents, *&c.* of angles than the angles them­selves, in trigonometrical calculations, the rules of which are to be deduced from the following theorems.

\*\*\*\*\*\*\*\*\*\*\*Τηεοrεμ I. In any right-

angled triangle, the radius is to the sine of either of the acute angles as the hypo­tenuse is to the side opposite that angle. (Fig· 2.)

Let ABC be a right-angled triangle, of which B is the right angle ; about C, one of the acute angles, as a centre, with a radius equal to the radius in the tables, describe an arc DE as the measure of the angle C (Def. 1). Draw DP, the sine of the arc DE (Def. 5). The triangles CDF, CAB are manifestly equiangular, there­fore their sides are proportionals ; so that CD : DF ≡ CA : AB, or (putting R to denote radius in the tables) R : sin. C = CA: AB.

*Corollary* 1. The radius is to the cosine of either of the oblique angles as the hypotenuse is to the side adjacent to that angle, l∙,or. to the radius CD, CP is the cosine of C, and CD : CF = CA : CB.

*CorMιu∕* 2. Assuming that CD = R = 1, we have

1 : sin.' C = CA : A B : and 1 : cos. C = CA : CB. Therefore, suj>posing the sides of the triangle expressed by numbers,

. r, AB CB

sui. C = ,-, and cos. C = \_ ..

**CA CA**

\*\*\*\*\*\*\*\*\*\*\*THEOREM II. In any right-angled triangle, the radius is to the tangent of either of the oblique angles as the adjacent side to the opposite side. (Fig. 2.)

At the point E, one extremity of the arc DE which mea­sures the angle C, draw EG perpendicular to CB ; then to the radius CE, EG is the tangent of the angle C (Def. 7). Now the triaiιg!es CEG, ABC being similar, CE : EG = CB : BA, or R : tan. C = CB : BA.

*corollary* 1. The radius is to the secant of either of the oblique angles as the adjacent side to the hypotenuse; for CG is the secant of the angle C (Def. 8) ; and CE : CG = CB : CΛ, or R : sec. C = CB : CA.

*corollary* 2. Assuming the radius = 1, then

. n V BA a *r>* CA tan. A = —, and sec. C = —.

tv 15 L 1>

\*\*\*\*\*\*\*\*\*\*f *οrοllary* 3. In any triangle, if a perpendicular be drawn from any one of the angles to t)>e opposite side, the seg­ments of that side are to one another as the tangents of the parts into which the opposite angle is divided by the per­pendicular. r

For in the triangles ADB, ADC,

BD : DA = tan. BAD : R; and DA : DC = R : tan. CAD. Therefore, *ex aquali,* BD : DC — tan. BAD ; tan. CAD.