By this rule, the values of the three, angles may be readily found, and also a verification of the calculation, be­cause the sum of the halves of the angles must make ninety degrees.·

Other solutions may be had from Theorems VII. VIII. IX. X. That from Theorem VII. (which resolves the triangle into two right-angled triangles by a perpendicular on one of its sides from the opposite angle) is very generally used by practical men not much versed in theory ; but the best solutions are those which follow from Theorems IX. X. XL These elegant formulae were first found about the year 1653, by William Purser of Dublin. The modifica­tion of Theorem XL, given here as a practical rule, is the best method we know for resolving this case.

*\*\*\*Examples of Oblique-angled Triangles.*

f side *a* = 575 j

\*\*\*Case I. Given angle B = 63o 48’ ( Required the

( angle C = 49 25 f sides Z>, *c.*

and therefore angle A = 66 47 )

Sin. A 9∙9C>333 Sin. A 9∙9f>333

Sin. B 9∙95292 Sin. C 9∙8805l

Side *a* 2∙75967 Side *a* 2∙75967

12-71259 12-64018

Side *b =* 56l∙4 2∙74926 Side *c* 475∙2 2’67685

\*\*\*Case II. Given P⅛e 6 = ) Required an-

fFiB∙7l ιs,d'' c = 2i2 .∙ glcs A, B, and

<rι8, '\*' ( angle C = 37o 20') side *a.*

Side *c* 2∙36549 From 180° 0'

Side *b* 2∙53782 subtract B -∣- C — 101 44

Sin. C 9∙78280

angle CAB = 78 16 12-32062

From 180 0

Sin.B= 64°∙2U I 9.9-51 subtract B’-f-C = 152 56 or sin. B'= 115 36 ∫ ,

angle CAB' = 27 4

In this example the angle A has two distinct values, from which we shall find two values of BC = *a,* and B'C = *a'.*

\*\*\*To find BC. To find B'C.

Sin. C 9-78280 Sin. C 9∙78280

Sin. B AC 9∙99083 Sin. B’AC 9∙65S04

Sidec 2∙36549 Side c 2\*36549

12-35632 12-02353

Side BC = o....2∙57352 Side B'C = β'....2∙24073

= 374∙6 ≡ 174∙ I

*\*\*\*Example* 2. In a plane triangle, there are given two sides, 532 and 358, and the angle opposite to the former, 107° 40' ; to find the remaining angles and side.

In this example, the things required have each only one

value. The angles will be found 39° 53' and 32° 27', and the remaining side 299∙6.

( side *b* — 1230 ] Requiret! angles

\*\*\*Case HI. Given - sido *c =x* 879 ,-B, C, and side ( angle A ≡ 105o ) *a.*

*b* + c = 2100, *b — c -* 360, B -f- C = 180° — A = 750, J,(B 4- C) = 370 30'

*b* + c 3∙32222 Cos. ( (B — C) 9∙99628

*b — c* 2∙5563O Cos. £ — (B + C) 9∙89947

Tan.j(B + C) = 37030' 9∙88498 *b+e* 8’32222

12-44128 13-22169

Tan.⅜(B-C)= 7 29⅛9∙ll906 «= 1680-4 8’22541

B = 44 59∣ C = 30 0∣

As a verification, *a* may be found from this other propor­tion :

Sin. ½ (B — C) : sin. ½ (B + C) = *b —* *c* : *a.*

The side *a* may also be found by Case I. in two ways, because all the angles and two sides are known.

20. In all the preceding operations, the logarithm of the fourth term of each proportion has been found by subtract­ing the logarithm of the first term from the sum of the lo­garithms of the second and third. Now, in every case, when one number *a* is to be subtracted from another number *b,* we may obtain the result required by adding the difference between *a*, and *n* any number, and then subtracting the num­ber *n* from the result. Thus, instead of subtracting 7 from 12, we may add 3, the difference between 7 and 10, to 12, and then rejecting 10 from 15, the sum, obtain 5 for the difference between 12 and 7. The reason of this may be easily understood, and it is evident from algebraic sub­traction ; for A — *a — b* + (n — *a*) — *n.* When *a*, the number to be subtracted, is less than 10, then *n* is conve­niently assumed = 10, and when *a* is between 10 and 20, then *n* may be taken = 20. Thus, if 17 is to be subtract­ed from 63, the remainder, 46, is found by adding 3 ( = 20 — 17) to 63, and rejecting 20 from the sum.

Hence, in trigonometrical calculation, instead of sub­tracting the log. of the first of four proportionals from the sum of the logs, of the second and third in order to obtain the log. of the fourth term, we may add into one sum the logs, of the second and third terms, and the difference be­tween the log. of the first and 10, and rejecting, mentally, 10 from the sum, we have what is required.

The difference between a logarithm and 10·00000, &c. is called the *arithmetical complement* of that logarithm ; and it is most readily obtained by subtracting every figure, be­ginning at the left hand, from 9, and the last significant figure from 10; thus, the arithmetical complement of 2·7365) is 7∙26349, and the arithmetical complement of 9∙78460 is 0·21540. By the introduction of the arithmeti­cal complements of logarithms, fewer figures are required in calculations ; for, with a little practice, it will be found as easy to read the arithmetical complements of logarithms from the table as the logarithms themselves. We have now this rule for finding a fourth proportional to three given numbers. Add together the arithmetical complement of the logarithm of the first term, and the logarithms of the second and third terms ; the sum is the logarithm of the fourth.

\*\*\*As an example, take the above proportion :

Sin. (B — C): sin. ½ (B + C) = *b — c* : *a.*

Sin. ⅛ (B — C) arith. comp, 0-b8465 . , .

Sin. ⅜ (B + C) 9∙78445

*b — c...................* 2\*55630

*a =* 1680-4 3∙22540

We have the same value of *a* as above.