25. There are six cases of oblique-angled triangles, which may be resolved by the formulæ which have been investigated ; but in some cases they will be modified so as to adapt them to logarithmic calculation.

Case I. Given the three sides *a, b, c,* to find the angles A, B, C.

Solution. \*\*\*\*\*\*\*Find s = ⅜ (α -∣- *b* -(- e),

and M = ∕-[ sin∙ <\* ~ ≡⅞⅛.-ftλgin∙ *<' ~s)*I ; then

*\*r (* Sin. 5 J

Tan. ⅜A = \* tan. ⅜B = .∙∙

a sin. (5 — *α) -* sin. ($ — *b)*

Tan.∣C=ςr-£ v

• sιn.(r — c)

This solution has been found in article 18.

Case II. Given two sides *a* and *b,* and the angle A op­posite.

(1.) The angle B may be found from formula II. thus:

\_. τ, sin. Asin.*b* Sm. B= i

sin. «

The affection of B is ambiguous, unless it can be de­termined by the rule A, 4- B is greater or less than 180o, according as *a -∣- b* is greater or less than 180° (art. 23).

(2.) The side *c* is to be found from this equation : [cos. *b*] cos. *e* -f- [sin. *b* cos. A] sin. *c —* cos. *a.*

Comparing this with the equation of art. 24, viz. M cos. *x* -f- N sin. *x =* P, we have

M = cos. *b,* N = sin. *b* cos. A, P = cos. *a ;* and following the analysis by which the angle *x* has been there determined, we find that if

Tan. p = tan. *b* cos. A,

, . . cos. *α* cos. a cos. *a* sin. ®

then cos. *(c — φ)= . r= . -. ζ.*

' cos.o sin. o cos. A

The first value of cos. *(c—* *φ*) being the more simple, is to be used. The arcs c — *φ* and *φ* being now both known, *c* is known.

\*\*\*\*\*3. To find the angle C, we have, from formula III., [cos. ⅛] cos. C -∣- [cot. A] sin. C = cot. *a* sin. *b.*

Here, referring to article 24, we have M *= cos. b,* N=cot. A, P ≡ cot. *a* sin. *b* ; therefore, assuming

cot. A Ian.©— z,

cos. *b*

we have

„ . , cot. *a* sin. *b*sin. a

Cos. (C — *φ) =* cot. *a* tan. *b* cos. f — -^∙ ∙

From either of these formulæ the angle C is dctcrmined.

Case III. Given the sides *a, b,* with the included angle 'C, to find the angles A, B, and side *c.*

(1.) The angles A, B, may be found from two of Na­pier’s analogies in article 22. Thu3 we have

T∙"l<Λ+≡) = oo∙-iC

τ\*"∙l(∙i-I>) = oo∙'iC⅛4⅛⅛J∙

Having now half the sum and half the difference of the angles A and B. each may be found by a known pro­cess. They may be otherwise found, by subsidiary angles, from formula I.

(2.) The side c is to be found from the equation [cos. a] cos. *b* -f- [sin. *a* cos. C] sin. 5 = cos. c.

Here we have M = cos. *a,* N = sin. *a* cos. C, P = cos. c ; we therefore make tan. *φ —* tan. *a* cos. C.

, . cos. c cos. *φ*

Then cos. *(b — φ) = ;*

' r cos. α

, , cos. *a* cos. *lb — a)*

and hence cos. *c —* -'r

cos. *f*

Case IV. Two angles A and B, and *c* the side between them, are given, to find the sides *a, b,* and angle C.

(1.) The sides *a, b* may be found from Napier’s formu­læ, article 22, thus,

Tan. J(α + fc) = tan. i c∙

Tan. ⅜ *(a \_ b) =* tan. ⅜ *c ■* g∙

Half t)ιc sum and half the difference of the sides *a, b* being found, each is thereby determined.

(2.) The angle C is found from the formula

[sin. A cos. c] sin. B — [cos. A] cos. B = cos. C. Here M = sin. A cos. *c,* N = cos. A, P = cos. C. We now assume

Cot. *φ* ≡ tan. A cos. *c.*

.,∙ι ■ /n , cos. C sin. *a*

1 hcn sin. (B — 0) = r—r-,

' r' cos. A

, „ cos. A sin. (B — *φ)*

and cos. C = ι—i

sin. *φ*

Case V. Two angles A, B and a side *a* opposite to one of them, are given, to find the sides *b, c,* and the angle C∙

(1.) The side *b* is found by this formula :

, sin. *a* sin. B Sin. *b = — 1—.*

sin. A

(2.) The side *c* is found by this formula :

[cot. π] sin. *c—* [cos. B] cos. c = cot. A sin. B.

Here M — cot. *a,* N = cos. B, P = cot. A sin. B ; we therefore assume

f, . cot. *a* Cot. ffi = e

cos. B’

and have sin. *(c— φ) —* cot A tan. B sin. *f.*

(3.) The angle C is found from the formula

[sin. B cos. λ] sin. C — [cos. B] cos. C = cos. A. Here M = sin. B cos. *a,* N ≡ cos. B, P = cos. A.

We assume cot. *φ =* tan. B cos. *a* ∏., . , . cos. A sin. β

lhen sin. *(c —* C) = κ—c.

' r' cos. B

Case VI. Given the three angles A, B, C, to find the sides *a, b, c.*

The solution of this case has been found in art. 19.

Find S = J (A + B + C),

and M,= ∕∫ co8∙ (S — A) cos. (S — B) cos.(S-C) ^> \*z I cos. S J ’

M, M'

Then cot. ⅛ *a =* zg rr, cot. ⅜ *b = - ε nλ,*

i cos. (S — A) \* (cos. b — B)

t . \_ M'

\* CO3. (S C)

These formulæ arc altogether analogous to those by which the angles are found from the sides. This case rarely occurs in the application of spherics to astronomy.

The analytical artifice of *subsidiary angles,* by which the formulæ are adapted to logarithmic calculation, corresponds to the resolving of an oblique spherical triangle into two right-angled triangles, by an arc of a great circle drawn from its vertex perpendicular to its base.

For the application of spherical trigonometry, see Practical Astronomy, chap. i. (vol. iv. page 63).

The treatises on trigonometry are innumerable. The most considerable is that of Cagnoli, in Italian, of which there is a French translation. On the history of trigono­metry, and the progress of its improvement, the reader will find the most ample information in Delambre, *Histoire de l'Astronomie Ancienne, Histoire de l'Astronomie du Moyen Age,* and *Histoire de l'Astronomie Moderne.* See also his *Astronomie Théorique et Pratique,* vol. i. chap. 10.