point in which the cycloid cuts it, is equal to \*\*— which shews that the curve and cycloid cut each other at right angles.

2. If the motion is supposed to commence at the same horizontal line, whatever he the point at which the body reaches the first curve; we have *h=d* constant, and the equation gives

\*\*\*γ1Λx1—γoΛrβ-∣-P∣ω∣—Poωo=0, from which it is evident, as in Problem 3, that the cycloid cuts both curves at right angles.

*<Iy*

3. If motion begins at the first curve, Λ=yrt and = -⅞=---i(--i=÷

hence our equation becomes

7∣δ\*1—γ0Ja∙,+ P 1ωl—P.ω0—*~ dx=Λ,*

*\*.*

or γ1⅛r1 + P1ω,=0, y,ta,+ P^>04-iy0(P1-P<,)=0.

From the first of these it follows that the lower curve is cut at right angles. The second gives

(γ.-P.P.)\*e.+P18y,=0, or c + pι^°=0∙ Also the first equation is c4-P1^l=O.

.∙. ; from which it appears that the tangents to

0Λ'0 o\*c∣

the two curves at the two points of section are parallel to one another.

10. We have hitherto solved only problems of absolute maxima and minima. But a very simple consideration will enable us to apply the same formulæ to the investigation of relative maxima and minima. The problem of isoperimetricals (prob. 5 below) will illustrate this class of ques­tions. Here the integrated function is not required to be a maximum or a minimum absolutely, but only one consistent with the further condition that another integrated function shall not change its value. Although, therefore, δ*∫ydx* is actually zero, yet δ*x* and δ*y* are not, as in our previous in­vestigations, *any* quantities whatever, but only such as will consist with the required condition that δ*∫*√1*+p2 dx* shall also be zero. To Euler we owe the idea of substituting for the quantity *u*, which is made to vary, not *fydx,* as in other cases, but *∫ydx+∫vdx∙, ∫νdx* being the quantity which is to remain unchanged. By this substitution we determine the relation between *x* and *y*, which renders *γ+av* a maximum or a minimum. But as that relation involves the arbitrary constant *a,* we restrict it by assign­ing to that quantity such a value as shall render *∫vdx* of the required magnitude. We have therefore found a relation, which not only makes the sum of two quantities a maximum or a minimum, but which likewise reduces one of those quan­tities to a specified value. This relation then evidently makes the other quantity a maximum or a minimum. Thus the problem is solved in all its generality.

*Prob.* 5. AB is a given line, PMQ a line perpendicular

to AB at the point M. The points P and Q are supposcd to generate two curves by the motion of the line perpen­dicular to AB, having a relation such, that although both are unknown, PM is a known function of QM. It is re­quired to determine both curves, when the length of AQB is given, and the area APBMA is a maximum.

Let QM=y, then PM=*f*(*y*), where *f* is a known fonc­tion.

Also length ABQ=*∫*√1+*p*2 *dx*, and area APB=*∫*f(*y*)*dx*, the limits of both integrals being the same.

\*\*\*\*\*∙∙∙ y=∕(y)+≡^Γ+pi>

, on\*

and by formula (4)∕(y) -∣- *a -J* 1 *+p,= ~f==i+c<*

*.J* 1 -pp θr ic-∕ Ο)} √ 1 *+p'=o,* the differential equation to the curve AQB.

Ify=φ(ar) be the solution of this equation, j=z∕'[ψ('r)j is the equation to the curve APB.

*Cor. lff(y)=y,* we get α\*=(c-y)\*4-(cz—a:)\*, the equa­tion to a circle.

*Prob.* 6. To find the curve which, *with a given length,* contains between its chord and its arc the greatest possible area.

By cor. to last problem the equation is α,=(c—*y),*

Let the origin be at A and AB=2r; then since y=0, both when *x=S)* and or=2r, we get

ÿ\*—*2cy~2rx—x'.*

And from the equation to the limits (γ1—Plp,)Λx1=.0, or γ1—P1p∣=0 .∙. *c—0* ; and the curve required is the semi­circle.

*Prob.* 7. Given the length of the curve contained be­tween two points in a horizontal line ; required its nature, that the centre of gravity of the arc may be the lowest possible. Here we have to make *fx>J* 1 *+pidx* a maxi­mum, whi1st ∕√l *+p, dx* is constant.

.∙. γ=ar√ 1 -∣-p,-∣-αV 1 *-∣∙p1,* and^\*=0, P=c,

*QX*

or x\*+⅛r.

√ 1 +p,

the integral of which is *y~c* log. (ar-}-α-∣-v∖.r-∣-αy—c\*) *+c,* the equation to the catenary.

The solution of more complicated problems is not con­sistent with our limits. For the investigation of formulæ in cases where *γ* involves an integral, or where it is given by a differential equation, the reader is referred to Woodhouse’s Treatise. Considerations on the mode of distin­guishing a maximum from a minimum will be found in Lagrange’s *Theorie des Fonctions Analytiques.*

Variation *of the compass.* See Magnetism.

Variation**,** in *Music,* means either the change induced upon a melody by embellishments given by the vocal or instrumental performer, or the new forms and more studied ornaments given by a composer to a theme of his own or to some popular air. Among instrumental variations, some of the most beautiful are to be found in Hadyn's andante movements, and in Mozart and Beethoven’s variations to their own andantes, or to popular themes on which they have displayed their skill and genius.

VARNA, a city of Turkey in Europe in the province of Silistra. It stands on the Black sea, at the mouth of the river of the same name. It is strongly fortified, and considered as one of the most important defences o∙' the southern di­vision of Turkey in Europe. The obstinate defence of it, in the late war with Russia, shows both its strength and its consequence. The harbour is good, and is the only one