is at the small crevice between the two metal shoulders. These shoulders need not touch, so that the friction must be insensible. We imagine that this method of tightening a turning joint may be used with great advantage in many cases.

We have only further to observe on this engine, that any imperfection by which the passage of the water is diminish­ed or obstructed produces a saving of water, which is in exact proportion to the diminution of effect. The only in­accuracy that is not thus compensated, is when the jets are not at right angles to the arms.

We repeat our wishes, that engineers would endeavour to bring this machine into use, seeing many situations where it may be employed to great advantage. Suppose, for in­stance, a small supply of water from a great height applied in this manner to a centrifugal pump, or to a hair belt pass­ing over a pulley, and dipping in the water of a deep well. This would be a hydraulic machine exceeding all others in simplicity and durability, though inferior in effect to some other constructions.

*Of Undershot Wheels.*

All wheels go by this name where the motion of the water is quicker than that of the partitions or boards of the wheel, and it therefore impels them. These are called the *float-boards,* or *floats,* of an undershot wheel. The water, run­ning in a mill-row, with a velocity derived from a head of water or from a declivity of channel, strikes on these floats, and occasions, by its deflections sidewise and upwards, a pressure on the floats sufficient for impelling the wheel.

There are few points of practical mechanics that have been more considered than the action of water on the floats of a wheel ; hardly a book of mechanics being silent on the subject. But the generality of them, at least such as are intelligible to persons who are not very much conversant in dynamical and mathematical discussion, have hardly done any thing more than copied the earliest deductions from the simple theory of the resistance of fluids. The consequence has been, that our practical knowledge is very imperfect ; and it is chiefly from experience that we must still learn the performance of undershot wheels. Unfortunately this stops their improvement ; because those who have the only op­portunities of making the experiments are not sufficiently acquainted with the principles of hydraulics, and are apt to ascribe differences in their performance to trifling nostrums in their construction, or in the manner of applying the im­pulse of the water.

We have said so much on the imperfection of our theories of the impulse of fluids in the article Resistance of Fluids, that we need not here repeat the defects of the common explanations of the motions of undershot wheels. The part of this theory of the impulse of fluids which agrees best with observation is, *that the impulse is in the duplicate proportion of the velocity with which the water strikes the float ;* that is, if *v* be the velocity of the stream, and *u* the velocity of the float, we shall have F, the impulse on the float when held fast, to its impulse *f* on the float moving with the velo­city *u,* as *v2* to (*v* — *u*)2, and *f* = \*\*\*F χ \*"∙

This is the pressure acting on the float, and urging the wheel round its axis. The wheel must yield to this motion, if the resistance of the work does not exert a superior pres­sure on the float in the opposite direction. By yielding, the float withdraws from the impulse, and this is therefore diminished. The wheel accelerates, the resistances increase, and the impulses diminish till they become an exact ba­lance for the resistances. The motion now remains uniform, and the momentum of impulse is equal to that of resistance. The performance of the mill therefore is determined by this ;

and, whatever be the construction of the mill, its perform­ance is best when the momentum of impulse is greatest. This is had by multiplying the pressure on the float by its velocity. Therefore the momentum will be expressed by

\*\*\*\*F × ——× «· But since F and *ν2* are constant quan­tities, the momentum will be proportional to *u × (v— u*)2*.* Let *x* represent the relative velocity. Then *v—x* will he = *u*, and the momentum will be proportional to *(v—x) × x*2*,* and will be a maximum when (*v*—*x)* × *x*2 is a maximum, or when *νx*2*—x*3 is a maximum. This will be discovered by making its fluxion = 0 ; that is,

2*νxdx—3x*2*dx* = 0

and 2*vx*— 3x2 = 0, or *2v—*3*x* = 0 and *2v = 3x,* and *x* = 2/3*v;* and therefore *υ — x,* or *u = 1/3υ.* That is, the velocity of the float must be one third of the velocity of the stream. It only remains to say what is the absolute pressure on the float thus circumstanced. Let the velocity *v* be supposed to arise from the pressure of a head of water *h.* The common theory teaches that the impulse on a given surface S at rest is equal to the weight of a column AS ; put this in place of F, and 4/9*v*2 in place of *(ν—u*)2 and 1/3*v* for *u.* This gives us S A × 4/27*v* for the momentum. Now the power expended is S*hv*, or the column S*h* moving with the velocity v. Therefore the greatest performance of an undershot wheel is equivalent to raising 4/27ths of the water that drives it to the same height.

But this is too small an estimation ; for the pressure exert­ed on a plane surface, situated as the float of a mill-wheel, is considerably greater than the weight of the column SA. This is nearly the pressure on a surface wholly immersed in the fluid. But when a small vein strikes a larger plane, so as to be deflected on all sides in a thin sheet, the impudse is almost double of this. This is in some measure the case in a mill-wheel. When the stream strikes it, it is heaped up along its face, and falls back again, and during this mo­tion it is acting with a hydrostatic pressure on it. When the wheel dips into an open river, this accumulation is less remarkable, because much escapes laterally ; but in a mill-course it may be considerable.

We have considered only the action on one float, but several generally act at once. The impulse on most of them must be oblique, and is therefore less than when the same stream impinges perpendicularly ; and this diminution of impulse is, by the common theory, in the proportion of the sine of the obliquity. For this reason it is maintained, that the impulse of the whole stream on the lowest float-board, which is perpendicular to the stream, is equal to the sum of the impulses made on all the floats which then dip into the water ; or that the impulse on any oblique float is precisely equal to the impulse which that part of the stream would have made on the lowest float-board had it not been interrupted. Therefore it has been recommended to make such a number of float-boards, that when one of them is at the bottom of the wheel, and perpendicular to the stream, the next in succession should be just entering into the water. But since the impulse on a float by no means annihilates all the motion of the water, and it bends round it and hits the one behind with its remaining force, there must be some advantage gained by employing a greater number of floats than this rule will permit. This is abundantly confirmed by the experiments of Smeaton and Bossut. The latter formed three or four suppositions of the number of floats, and calculated the impulse on each, according to the ob­servations made in a course of experiments by the Aca­demy of Sciences, and inserted by us in the article Resistance of Fluids; and when he summed them up, and compared the results with his experiments, he found the agreement very satisfactory. He deduces a general rule,