the wave-crest, as viewed against the side of the ship, is constantly diminishing. We see the wave-crest is almost at right angles to the ship, but the outer end is slightly deflected sternward from the

circumstance that when a wave is entering undisturbed water its progress is a little retarded, and it has to deflect itself into an oblique position, so that its oblique progress shall enable it exactly to keep pace with the ship. The whole wave-making resistance is the resistance expended in generating first the diverging bow waves, which, as we have seen, cease to act on the ship when once they have rolled clear of the bow ; secondly, these transverse waves, the crests of which remain in contact with the ship’s side ; and thirdly, the terminal wave, which appears independently at the stern of the ship. This latter wave arises from causes similar to those which create the bow wave, namely, the pressure of the streams which, forced into divergence then, here converge under the run of the vessel, and re-establish an excess of pressure at their meeting. The term ‘ wave-making resistance ' represents, then, the excess of resistance beyond that due to surface friction, and that excess we know to be chiefly due to this formation of waves by the ship.”

Pursuing these experiments it was found that not only was there a certain length of form necessary in a ship designed to attain a certain speed economically,—a fact which Mr Scott Russell did much to establish,—but that there was also a considerable increase in wave-making resistance dependent upon the position of the after­body or run of the ship with reference to the wave-system left by the bow. Stating this again in Mr Froude’s words :—

“The waves generated by the ship in passing through the water originate in the local differences of pressure caused in the surrounding water by the vessel passing through it ; let us suppose, then, that the features of a particular form are such that these differences of pressure tend to produce a variation in the water level shaped just like a natural wave, or like portions of a natural wave of a certain length.

“ Now an ocean wave of a certain length has a certain appropriate speed at which only it naturally travels, just as a pendulum of a certain length has a certain appropriate period of swing natural to it. And, just as a small force recurring at intervals corresponding to the natural period of swing of a pendulum will sustain a very large oscillation, so, when a ship is travelling at the speed naturally appropriate to the waves which its features tend to form, the stream line forces will sustain a very large wave. The result of this phenomenon is, that as a ship approaches this speed the waves become of exaggerated size, and run away with a proportionately exaggerated amount of power, causing corresponding resistance. This is the cause of that very disproportionate increase of resistance experienced with a small increase of speed when once a certain speed is reached.

“We thus see that the speed at which the rapid growth of resistance will commence is a speed somewhat less than that appropriate to the length of the wave which the ship tends to form. Now, the greater the length of a wave is the higher is the speed appropriate to it; therefore the greater the length of the waves which the ship tends to form the higher will be the speed at which the wave-making resistance begins to become formidable. We may therefore accept it as an approximate principle that the longer are the features of a ship which tend to make waves the higher will be the speed she will be able to go before she begins to experience great wave-making resistance, and the less will be her wave-making resistance at any given speed. This principle is the explanation of the extreme importance of having at least a certain length of form in a ship intended to attain a certain speed ; for it is necessary, in order to avoid great wave-making resistance, that the ‘wave features,’ as we may term them, should be long in comparison with the length of the wave which would naturally travel at the speed intended for the ship.

“ This view of the matter, then, recognizes the tendency of a ship, when the speed bears a certain relation to the length of. her wave-making features, to make largo waves and to incur correspond­ing wave-making resistance. But it does not take account of the possibility of the waves made by one feature of the form so placing themselves with reference to other features as, by the differences of pressure essential to their existence, either to cause an additional resistance, or on the other hand to cause a forward force which partly counterbalances the resistance originally due to their creation. The way in which this may occur we have seen strikingly exhibited in the results of the experiments I have been describing. We see that in the very long parallel-sided form the sternmost of the train of waves left by the bow has become so small that its effect on the stern is almost insensible ; and here we find, consequently, the united resistance due simply to the generation of a separate wave-system by each end of the ship. As we gradually reduce the length of middle-body, the stern is brought within the reach of waves large enough to produce a sensible effect, and according as it is brought into conjunction with a crest or hollow, the total wave-making resistance becoming least of all (except at the very highest speed) when the middle-body is reduced to nothing.”

The variations in residuary resistance due to these transverse wave-formations are variations of quasi-hydrostatic pressure against the after-body, corresponding with the changes in its position with reference to the phases of the train of waves, there being a com­parative excess of pressure (causing a forward force or diminution of resistance) when the after-body is opposite a crest, and the reverse when it is opposite a trough.

It may be proper to introduce here some remarks as to the stream lines which have been referred to in the foregoing considerations. The statement of the case as given by Mr Froude, and derived by him mainly from the investigations of Prof. Rankine, is as follows :—

“By a ‘perfect fluid’ is meant one the displacements of which are governed solely by the laws expressed in the equation of fluid motion, the particles of which therefore are without viscosity, and are capable of gliding rectilinearly along a perfectly smooth surface or past each other without frictional interference. By an imperfect fluid is meant one in which, as in water, as well as those with which we are practically acquainted, such frictional interference is inevitable.

“ Dealing first, then, with the case of steady rectilinear motion in a perfect incompressible fluid, infinitely extended in all directions, it is plain that the motion will create differences of pressure, and therefore changes of velocity, in the particles of the surrounding fluid, which thus move in what are called ‘stream lines.’ At the commencement of the motion of the body the particles of the fluid undergo acceleration in their respective stream-line paths, and these accelerations imply a resistance experienced by the body ; but after the motion has become established the differences of pressure satisfy themselves by keeping up the stream-line con­figuration ; the energy which the particles receive from the body while they are being pushed aside by it along their stream-line paths is finally redelivered by them to it as they collapse around it, and come to rest after its passage, and the integrals of the + and - pressures on the body are exactly equal at every moment. The manner in which this is effected is governed by the general laws of fluid motion, as expressed by the well-known equations ; and, since these equations contain no term which implies a loss of energy, the energy existing in the body, as well as in the stream­line system, remains unaltered ; so that, if the motion is steady, or without acceleration or retardation, the body passes through this theoretically perfect fluid absolutely without resistance. Nor must it be thought a paradox (for it is unquestionable) that even a plane moving steadily at right angles to itself through a perfect fluid would in the manner described experience no resistance. But if the fluid, instead of being infinite in all directions, be bounded by a definite free surface parallel to the line of motion, such as a water level, the existence of this surface cuts off the reactions of all those particles which would have existed beyond the surface had the fluid been unlimited alike in all directions, and which would have given back in the manner described the energy imparted to them. By the absence of these reactions the stream-line motions which would have existed in the infinite fluid are modified, and the differences of pressure involve corresponding local eleva­tions of the surface of the water in the vicinity of the moving body. And since, in consequence of the action of gravitation (the force which controls the surface), a water protuberance seeks immediately to disperse itself into the surrounding fluid in accordance with the laws of wave motion, the local elevation partly discharges itself along the surface by waves which carry with them the amount of energy embodied in their production. This energy is, in fact, part of the aggregate energy which was imparted to the particles of fluid while they were being pushed aside, and which, in the infinitely extended fluid, would have been wholly restored to the body during their collapse after its passage, but is now, in fact,