IV. Iron bar bent by supporting at one end, the other end being loaded ; alternations of stress from pull to push caused by rotating the bar :—

|  |  |  |  |
| --- | --- | --- | --- |
| Stress.  From + to — | Number of Rotations causing Rupture. | Stress.  From + to — | Number of Rotations causing Rupture. |
| 320 | 56,430 | 220 | 3,632,588 |
| 300 | 99,000 | 200 | 4,917,992 |
| 280 | 183,145 | 180 | 19,186,791 |
| 260 | 479,490 | 160 | Not broken with 1321/4 |
| 210 | 909,810 |  | millions. |

46. From these and other experiments Wohler concluded that the wrought-iron to which the tests refer could probably bear an indefinite number of stress changes between the limits stated (in round numbers) in the following table (the ultimate tensile strength was about 191/2 tons per square inch) :—

|  |  |
| --- | --- |
|  | Stress in Tons per Sq. Inch. |
| From pull to push |  |
| From pull to no stress | 13 to 0 |
| From pull to less pull | 19 to 101/2 |

Hence it appears that the actual strength of this material varies in a ratio which may be roughly given as 3 : 2 : 1 in the three cases of (*a*) steady pull, (*b*) pull alternating with no stress, very many times repeated, and (*c*) pull alternating with push, very many times repeated. Factors of safety applicable to the three cases might therefore rationally stand to one another in the ratio of 1 : 2 : 3. For steel Wohler obtained results of a generally similar kind. His experiments were repeated by Spangenberg, who extended the inquiry to brass, gun-metal, and phosphor-bronze.@@1 On the basis of Wohler’s results formulas have been devised by Launhardt, Weyrauch, and others to express the probable actual strength of metals under assigned variations of stress ; these are, of course, of a merely empirical character, and the data are not yet extensive enough to give them much value.@@2

47. Wohler’s experiments, dealing, as all experiments must, with a finite number of stress-changes, leave it an open question whether there are any limits within which a state of stress might be indefinitely often varied without finally destroying the material. It is natural to suppose that a material possessing perfect elasticity would suffer no deterioration from stress-changes lying within limits up to which the elasticity is perfect. But these limits, if they exist at all, are probably very narrow. Indeed, in the case of iron, there is indirect evidence that all alteration of stress whatso­ever affects the molecular structure in a way not consistent with the notion of perfect elasticity. When the state of stress in iron is varied, however slowly aud however little, the magnetic and thermo-electric qualities of the metal are found to change in an essentially irreversible manner.@@3 Every variation leaves its mark on the quality of the piece ; the actual quality at any time is a function of all the states of stress in which the piece has previously been placed. It can scarcely be doubted that sufficiently refined methods of experiment would detect a similar want of reversibility in the mechanical effects of stress, even when alterations of stress occur slowly enough to escape the effects of viscosity which have been examined by Sir William Thomson and discussed under Elasticity (vol. vii. pp. 802 *sq. ).* In any case, the viscosity investi­gated by Thomson causes such stress-changes as occur rapidly to do work on the material, and the destructive effect of repeated changes may be due in great part to this cause. His experiments show that rapid stress-changes often repeated do produce a cumulative effect in reducing the modulus of elasticity; and it is very probable that this fatigue of elasticity is associated with fatigue of strength.

There are as yet no experiments showing how far fatigue of strength is affected by the frequency, as distinguished from the mere number, of the stress-changes, nor whether a period of rest, after fatigue has been induced, restores strength. That it does so may be conjectured from Thomson’s discovery that rest restores elasticity after elastic fatigue. The conjecture is strengthened by Bauschinger’s discovery that, after a permanent set has been pro­duced and a period of rest follows, the apparent limit of elasticity (in the strict sense of that term) rises slowly with the lapse of time. Both questions are of obvious practical interest.@@4

48. When a strain is produced within the limits to which Hooke’s law applies, the work done in producing it is half the product of the stress into the strain. A load applied to a piece suddenly, but without impact, does an amount of work in straining the piece which is measured by the weight of the load into the distance it sinks in consequence of the strain. Hence, provided

this strain falls within the elastic limit, the strain and the stress are twice as great as the same load would produce when in equi­librium. Instances of load applied with complete suddenness, and yet without shock, are rare ; but it is a common case for loads to be applied so rapidly that the stress reaches a value intermediate between that due to a static load and the double stress due to the same load applied at once. Thus the Railway Commissioners found that certain bridges were deflected by a train passing at a speed of 50 miles per hour 1/7 more than by the same load at rest.@@5 The fact that a “live” load produces greater stress than a dead load is of course to be distinguished from the question Wohler’s experi­ments deal with—the greater destructiveness of the intermitted or varied stress which a live load causes. In many cases engineers allow in one operation for these quite independent influences of a live load by choosing a higher factor of safety for the live than for the dead part of the whole load on a structure, or (what is the same thing) by multiplying the live load by a coefficient (often 11/2), adding the product to the dead load, and treating the sum as if all were dead load.

49. A useful application of diagrams showing the relation of strain to stress is to determine the amount of work done in strain­ing a piece in any assigned way. The term “resilience” is conven­iently used to specify the amount of work done when the strain just reaches the corresponding elastic limit. Thus a rod in simple tension or simple compression has a resilience per unit of volume =*f*2∕2E, where *f* is the greatest elastic pull or push. A blow whose energy exceeds the resilience (reckoned for the kind of stress to which the blow gives rise) must in the most favourable case pro­duce a permanent set; in less favourable cases local permanent set will be produced although the energy of the blow is less than the resilience, in consequence of the strain being unequally dis­tributed. In a plastic material a strain exceeding the limit of elasticity absorbs a relatively large amount of energy, and generally increases the resilience for subsequent strains. Fracture under suc­cessive blows, as in the testing of rails by placing them as beams on two supports, and allowing a weight to fall in the middle from a given height, results from the accumulated set which is brought about by the energy of each blow exceeding the resilience

50. In an important paper@@6 which is reprinted in the article Elasticity, and should be carefully studied in this connexion, Prof. James Thomson has pointed out that the effect of any externally applied load depends, to a very material extent, on whether there is or is not initial internal stress, or, in other words, whether the loaded piece is initially in what Prof. Hearson has called a state of ease. Internal stress, existing without the application of force from without the piece, must satisfy the con­dition that its resultant vanishes over any complete cross-section. It may exist in consequence of set caused by previously applied forces (a case of which instances are given below), or in conse­quence of previous temperature changes, as in cast-iron, which is thrown into a state of internal stress by unequally rapid cooling of tho mass. Thus in (say) a spherical casting an outside shell solidifies first, and has become partially contracted by cooling by the time the inside has become solid. The inside then contracts, and its contraction is resisted by the shell, which is thereby compressed in a tangential direction, while the metal in the interior is pulled in the direction of the radius. Allusion has already been made to the fact, pointed out by J. Thomson, that the defect of elasticity under small loads which Hodgkinson discovered in cast-iron is probably due to initial stress. In plastic metal a nearly complete state of ease is brought about by annealing ; even annealed pieces, however, sometimes show, in the first loading, srnall defects of elasticity, which are probably due to initial stress, as they disappear when the load is reapplied.

51. Little is exactly known with regard to the effect of tempera­ture on the strength of materials. Some metals, notably iron or steel containing much phosphorus, show a marked increase in brittle­ness at low temperatures, or “cold shortness.” Experiments on the tensile strength of wrought-iron and steel show in general little variation within the usual atmospheric range of heat and cold. The tensile strength appears to be slightly reduced at very low temperatures, and to reach a maximum when the metal is warmed to a temperature between 100° C. and 200° C. When the tempera­ture exceeds 300° C. the tensile strength begins to fall off rapidly, and at 1000° C. it is less than one-tenth of the normal value.@@7 Reference may be made, in this connexion, to the effect which a “ blue heat,” or temperature short of red heat, is believed to have on the plasticity and strength of iron, and more especially of mild steel. It appears that steel plates and bars bent or otherwise

@@@1 *Ueber das Verhalten der Metalle bei weiderholten Anstrengungen,* Berlin, 1875.

@@@2 See Weyrauch, “ Οn the Calculation of Dimensions as depending on the Ultimate Working Strength of Materials," *Min. Proc. Inst. C.E,,* vol. lxiii. p. 275; also a correspondence in *Engineering,* vol. xxix., and Unwin's *Machine Design,* chap. ii.

@@@3 Ewing, *Phil. Trans.,* 1885, 1886.

@@@4 For interesting notices of the fatigue of metals in railway axles, bridge ties, &c., and results of experiments showing reduced plasticity in fatigued metal, see Mr B. Baker’s address to the Mechanical Section of the British Association, 1885. In most cases where the fatigue of metals occurs in engineering practice the phenomenon is complicated by the occurrence of blows or shocks whose energy is absorbed in producing strains often exceeding the elastic limits, sometimes of a very local character in consequence of the inertia of the strained pieces. Such shocks may cause an accumulation of set which finally leads to rupture in a way that is not to be confused with ordinary fatigue of strength. It appears that the effects of fatigue may be removed by annealing.

@@@5 *Report of Commissioners on the Application of Iron to Railway Structures,* 1849. A mathematical investigation of this effect of rolling load is given in an appendix to the Report.

@@@6 *Camb. and Dub. Math. Journ.,* Nov., 1848.

@@@7 See *Report of a Committee of the Franklin Institute,* 1837 ; Fairbairn, *Brit. Ass.* *Rep.,* 1856 ; Styffe on *Iron and Steel,* trans. by C. P. Sandberg. Notices of these and other experiments will be found in Thurston's *Materials of Engineering,* ii. chap. x., and in papers by J. J. Webster, *Min*. *Proc. Inst. C.E.,* vol. lx,, and A. Martens, *Zeitschr. des Ver. Deutsch. Ing.,* 1883.