to break the beam (M1) cannot be calculated from the ultimate tensile or compressive strength of the material by using the for­mula M1 =*f*cI∕*y*1, or *f*cI∕*y*2. When experiments are made on the ultimate strength of bars to resist bending, it is not unusual to apply a formula of this form to calculate an imaginary stress *f* which receives the name

of the modulus of trans­

verse rupture. Let the

section be such that y1 =

y2. Then the modulus

of transverse rupture is

defined as *f*= M1*y*1∕I.

This mode of stating the

results of experiment on

transverse strength is

unsatisfactory, inasmuch

as the modulus of rup­

ture thus determined

will vary with different

forms of section. Thus a plastic material for which *f*c and *f*t are equal, if tested in the form of an I beam in which the flanges form practically the whole area of section, will have a modulus of rapture sensibly equal to *f*t or *fc,* On the other hand, if the material be tested in the form of a rectangular bar, the modulus of rupture may approach a value one and a half times as great. For in the latter case the distribution of stress may approach an ultimate con­dition in which half the section is in uniform tension *ft,* and the other half in uniform compression of the same intensity. The mo­ment of stress is then 1/4 *ftbh2, b* being the breadth and *h* the depth of the section ; but by definition of the modulus of rupture *f,* M=1/6*fbh*2. In tables of the modulus of transverse rupture the values are generally to be understood as referring to bars of rectangular section. Values of this modulus for some of the principal materials of engineering are given in the article Bridges, vol. iv. p. 292.

59. The strain produced by bending stress in a bar or beam is, as regards any imaginary filament taken along the length of the piece, sensibly the same as if that filament were directly pulled or compressed by itself. The resulting deformation of the piece consists, in the first place and chiefly, of curvature in the direction of the length, due to the longitudinal extension and compression of the filaments, and, in the second place, of transverse flex­ure, due to the lateral compression and extension which go along with their longitudinal extension and com­pression (see Elasticity, § 57). Let *l*1, fig. 33, be a short portion of the length of a beam strained by a bend­ing moment M (within the limits of elasticity). The beam, which we assume to be originally straight, bends in the direction of its length to a curve of radius R, such that R∕Z = y1∕*δl*, *δl* being the change of *l* by exten­sion or compression, at a distance *y*1 from the neutral axis. But *δl*=*lp*1∕E by § 10, and *p*1=M*y*1∕I. Hence R = EI∕M. The transverse flexure is not, in general, of practical importance. The centre of curvature for it is on the opposite side from the centre for longitudinal flexure, and the radius is Rσ, where *σ* is the ratio of longitudinal extension to lateral contraction under simple pull.

60. Bending combined with shearing is the mode of stress to which beams are ordinarily subject, the loads, or externally applied forces, being applied at right angles to the direction of the length. Let AB, fig. 34, be any cross-section of a beam in equilibrium. The portion V of the beam, whieh

lies on one side of AB, is in equi­

librium under the joint action of

the external forces F1, F2, F3, &c.,

and the forces which the other por­

tion U exerts on V in consequence

of the state of stress at AB. The

forces F1, F2, F3, &c. may be referred

to AB by introducing couples whose

moments are F1*x*1, F2*x*2, F3*x*3, &c. Hence the stress at AB must equilibrate, first, a couple whose moment is ΣF*x*, and, second, a force whose value is 2F, which tends to shear V from U. In these summations regard must of course be had to the sign of each force ; in the diagram the sign of F3 is opposite to the sign of F1 and F2, Thus the stress at AB may be regarded as that due to a bending moment M equal to the sum of the moments about the section of the externally applied forces on one side of the section (ΣF*x*), and a shearing force equal to the sum of the forces about one side of the section (2F). It is a matter of convenience only whether the forces on V or on U be taken in reckoning the bending moment and the shearing force. The bending moment causes a uniformly vary­ing normal stress on AB of the kind already discussed in § 56 ; the shearing force causes a shearing stress in the plane of the section, the distribution of which will be investigated later. This shearing stress in the plane of the section is (by § 6) accompanied by an

equal intensity of shearing stress in horizontal planes parallel to the length of the beam.

61. The stress due to the bending moment, consisting of longi­tudinal push in filaments above the neutral axis and longitudinal pull in filaments below the neutral axis, is the thing chiefly to be considered in practical problems relating to the strength of beams. The general formula *p*1 = M*y*1∕I becomes, for a beam of rectangular section of breadth *b* and depth *h*, *p*1 = 6M∕*bh*2 = 6M∕SA, S being the area of section. For a beam of circular section it becomes *p*1 = 32M∕π*h*3 = 8M*∕Sh.* The material of a beam is disposed to the greatest advantage as regards resistance to bending when the form is that of a pair of flanges or booms at top and bottom, held apart by a thin but stiff web or by cross-bracing, as in **I** beams and braced trusses. In such cases sensibly the whole bending moment is taken by the flanges; the intensity of stress over the section of each flange is very nearly uniform, and the areas of section of the ten­sion and compression flanges (S1 and S2 respectively) should be proportioned to the value of the ultimate strengths *ft* and *fc,* so that S1*ft*=S2 *fc*. Thus for cast-iron beams Hodgkinson has recommended that the tension flange should have six times the sectional area of the compression flange. The intensity of longitudinal stress on the two flanges of an Ibeam is approximately M∕S1*h* and M∕S2*h*, *h* being the depth from centre to centre of the flanges.

62. In the examination of loaded beams it is convenient to re­present graphically the bending moment and the shearing force at various sections by setting up ordinates to represent the values of these quantities. Curves of bending moment and shearing force for a number of important practical cases of beams supported at the ends will be found in the article Bridges, with expressions for the maximum bending moment and maximum shearing force under various distributions of load. The subject may be briefly illustrated

here by taking the case of a cantilever or projecting bracket—(1) loaded at the end only (fig. 35) ; (2) loaded at the end and at another point (fig. 36) ; (3) loaded over the whole length with a uniform load per foot run. Curves of bending moment are given in full lines and curves of shearing force in dotted lines in the diagrams.

The area enclosed by the curve of shearing force, up to any ordinate, such as *ab* (fig. 37), is equal to the bending moment at the same section, represented by the ordinate *ac.* For let *x* be increased to *x + δx,* the bending moment changes to 2F(*x* + δ*x*), or δM = δ*xΣ*F. Hence the shearing force at any section is equal to the rate of change of the bending moment there per unit of the length, and the bending moment is the integral of the shearing force w’ith respect to the length. In the case of a continuous dis­tribution of load, it should be observed that, when *x* is increased to *x*+δ*x*, the moment changes by an additional amount W'hich depends on (δx)2 and may therefore be neglected.

63. To examine the distribution of shearing stress over any vertical section of a beam, we may consider two closely adjacent sections AB and DE (fig. 38), on which the bending mo­ments are M and M + δM respectively. The result­ant horizontal force due to the bending stresses on a piece ADHG enclosed be­tween the adjacent sec­tions, and bounded by the horizontal plane GH at a distance *y*0 from the neutral axis, is shown by the shaded figure. This must be equilibrated by the horizontal shearing stress on GH, which is the only other horizontal force acting on the piece. At any height *y* the intensity of resultant horizontal stress due to the difference of the bending moments is *y*δM∕I, and the whole horizontal force

on GH is *⅛lfPvιyzdy, z* being the breadth. If *q* be the intensity

of horizontal shearing stress on the section GH, whose breadth is *z*0, we have

*qzjδx=^-∕ yzdy .*

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