*Orthotomic Surfaces; Parallel Surfaces.*

20. The three sets of surfaces may be such that the

three surfaces through any point of space whatever inter­sect each other at right angles; and they are in this case said to be orthotomic. The term curvilinear coordinates was almost appropriated by Lamé, to whom this theory is chiefly due, to the case in question : assuming that the equations *p=f*1(*x, y, z*), *q=f*2(*x, y, z*), *r=f*3(*x, y, z*), refer to a

system of orthotomic surfaces, we have in the restricted sense *p,* *q, r* as the curvilinear coordinates of the point.

An interesting special case is that of confocal quadric surfaces. The general equation of a surface confocal with the ellipsoid -+^+3.lfa - + ^+-^-l∙

and, if in this equation we consider *x, y, z* as given, we have for *θ* a cubic equation with three real roots *p, q, r,* and thus we have through the point three real surfaces, one an ellipsoid, one a hyperboloid of one sheet, and one a hyperboloid of two sheets.

21. The theory is connected with that of curves of cur­vature by Dupin’s theorem. Thus in any system of ortho­tomic surfaces each surface of any one of the three sets is intersected by the surfaces of the other two sets in its curves of curvature.

22. No one of the three sets of surfaces is altogether arbitrary: in the equation *p=f*1(*x, y, z*), *p* is not an arbi­trary function of *x, y, z,* but it must satisfy a certain partial differential equation of the third order. Assuming that *p* has this value, we have *q = ffx, y, z)* and *r = f3(x, y, z)* determinate functions of *x,y,z* such that the three sets of surfaces form an orthotomic system.

23. Starting from a given surface, it has been seen (No. 16) that the normals along the curves of curvature form two systems of torses intersecting each other, and also the given surface, at right angles. But there are, intersecting the two systems of torses at right angles, not only the given surface, but a singly infinite system of surfaces. If at each point of the given surface we measure off along the normal one and the same distance at pleasure, then the locus of the points thus obtained is a surface cutting all the normals of the given surface at right angles, or, in other words, having the same normals as the given surface; and it is therefore a parallel surface to the given surface. Hence the singly infinite system of parallel surfaces and the two singly infinite systems of torses form together a set of orthotomic surfaces.

*The Minimal Surface.*

24. This is the surface of minimum area—more ac­curately, a surface such that, for any indefinitely small closed curve which can be drawn on it round any point, the area of the surface is less than it is for any other surface whatever through the closed curve. It at once follows that the surface at every point is concavo-con­

vex ; for, if at any point this was not the case, we could, by cutting the surface by a plane, describe round the point an indefinitely small closed plane curve, and the plane area within the closed curve would then be less than the area of the element of surface within the same curve. The condition leads to a partial differential equa­tion of the second order for the determination of the minimal surface : considering *z* as a function of *x, y,* and writing as usual *p, q, r, s, t* for the first and second differential coefficients of *z* in regard to *x, y* respectively, the equation (as first shown by Lagrange) is (1+*q*2)r - 2*pqs* + (1+*p*2)*t* = 0, or, as this may also be written,

*f- ■ .* + 4~ = = 0. The general integral

*dy* √1+7y2 + 72^f⅛ √1+j02 + i2

contains of course arbitrary functions, and, if we imagine these so determined that the surface may pass through a given closed curve, and if, moreover, there is but one minimal surface passing through that curve, we have the solution of the problem of finding the surface of minimum area within the same curve. The surface continued be­yond the closed curve is a minimal surface, but it is not of necessity or in general a surface of minimum area for an arbitrary bounding curve not wholly included within the given closed curve. It is hardly necessary to remark that the plane is a minimal surface, and that, if the given closed curve is a plane curve, the plane is the proper solution ; that is, the plane area within the given closed curve is less than the area for any other surface through the same curve. The given closed curve is not of necessity a single curve : it may be, for instance, a skew polygon of four or more sides.

The partial differential equation was dealt with in a very remarkable manner by Riemann. From the second

form given above it appears that we have *~~-JyL≈~~* ∖ 1 +7>2 + ç2

= a complete differential, or, putting this = *dζ,* we intro­duce into the solution a variable *ζ,* which combines with 3 in the forms *z±iζ (i=* √ - 1 as usual). The boundary conditions have to be satisfied by the determination of the conjugate variables *η, η'* as functions of *z + iζ, z - iζ,* or, say, of *Z, Z'* respectively, and by writing *S, S'* to denote *x + iy, x - iy* respectively. Riemann obtains finally two ordinary differential equations of the first order in *S, S', η, η', Z, Z',* and the results are completely worked out in some very interesting special cases.

The memoirs on various parts of the general subject are very numerous ; references to many of them will be found in Salmon’s *Treatise on the Analytic Geometry of Three Dimensions,* 4th ed., Dublin, 1882 (the most comprehensive work on solid geometry) ; for the minimal surface (which is not considered there) see Memoirs xviii. and xxvi. in Riemann’s *Gesammelte mathematische Werke,* Leipsic, 1876 ; the former—“Ueber die Fläche vom kleinsten Inhalt bei gegebener Begrenzung,” as published in *Gott. Abhandl.,* vol. xiii. (1866-67)—contains an introduction by Hattendorff giving the history of the question. (A. CA.)

SURGEONS, College of. See Societies.

SURGERY

Part I.—History.

Surgery in all countries is as old as human needs. A certain skill in the stanching of blood, the extraction of arrows, the binding up of wounds, the supporting of broken limbs by splints, and the like, together with an in­stinctive reliance on the healing power of the tissues, has been common to men everywhere. In both branches of the Aryan stock surgical practice (as well as medical) reached a high degree of perfection at a very early period. It is a matter of controversy whether the Greeks got their medicine (or any of it) from the Hindus (through the medium of the Egyptian priesthood), or whether the Hindus owed

that high degree of medical and surgical knowledge and skill which is reflected in Charaka and Susruta (commenta­tors of uncertain date on the Yajur-Veda; see Sanskrit, vol. xxi. p. 294) to their contact with Western civilization after the campaigns of Alexander. The evidence in favour of the former view is ably stated by Wise in the Intro­duction to his *History of Medicine among the Asiatics* (London, 1868). The correspondence between the *Susruta* and the *Hippocratic Collection* is closest in the sections relating to the ethics of medical practice ; the description, also, of lithotomy in the former agrees almost exactly with the account of the Alexandrian practice as given by Celsus. But there are certainly some dexterous operations described