they originated. For this purpose assume *X* to be the angle opposite the flank side of any triangle, and *Y* and *Z* the angles opposite the sides of continuation ; also let *x, y,* and *z* be the most probable values of the errors of the angles which will satisfy the given equations of condition. Then each equation may be expressed in the form *[ax+by + cz] = E,* the brackets indicating a summation for all the triangles involved. We have first to ascertain the values of the coefficients *a, b,* and *c* of the unknown quantities. They are readily found for the side equations on the circuits and between the base-lines, for *x* does not enter them, but only *y* and *z,* with coefficients which are the cotangents of *Y* and *Z,* so that these equations are simply [cot *Y.y* - cot *Z.z*] *= E.* But three out of four of the circuit equations are geodetic, corresponding to the closing errors in latitude, longitude, and azimuth, and in them the co­efficients are very complicated. They are obtained as follows. The first term of each of the three expressions for ∆λ, ΔA, and *B* is differentiated in terms of *c* and *A,* giving

*d.∆λ =* ∆λ ( *—-dA* tanH sin 1" j-

*d.ΔL= ΔB*I *c~+dA* cot *A* sin 1" j- - (15),

*dB = dA + ΔA ( “C + dA* cot *A* sin 1" j-

in which *dc* and *dA* represent the errors in the length and azimuth of any side c which have been generated in the course of the triangulation up to it from the base-line and the azi­muth station at the origin. The errors in the latitude and longitude of any station which are due to the triangula­tion are *d*λ, = [*d*.∆λ], and *dL,* = [*d*. Δ*L*].

Let station 1 be the origin, and let 2, 3, . .. be the succeeding stations taken along a predetermined line of traverse, which may either ran from vertex to vertex of the successive tri­angles, zigzagging between the flanks of the chain, as in fig. 3 (1), or be carried directly along one of the flanks, as in fig. 3 (2). For the general sym­bols of the differential equations sub­stitute ∆λ*n*, Δ*L*n, *ΔAn, cn, An,* and *Bn,* for the side between stations *n* and *n* + 1 of the traverse ; and let *δcn* and *δAn* be the errors generated between the sides cn-1 and *cn ;* then

⅛Cj-δcji *de2-δci* + δc2. <fc„\_ "Γδc-]

cl cl *c2* C1 c2 *Cn* lLc J

*dA1 = δAi* j *dA2 = dB1 + δΑ2; ... dAn = dΒn~i + δAn.*

Performing the necessary substitutions and summations, we get

f J∆^⅛ + J∆∠⅛ + . . . +Δ∠⅛

<LBn={ *n ^ 2 n*

I +(1+ 1[∆ricotri]sinl")δ√f1 + (l + .j[∆ri cot√∕]sinl")δ√i2 ∖ + . . . + (1+ ∆rincot √fnsin l")δrin.

∩[ΔX]⅛ + "[Δλ]⅛+...+Δλll^

I <-l t2 tn

*(I — ∙∖ ∏*

- {1[∆λ tan *A]δA1* + J∆λ tan *A]δA2 + ...*

∖ + ∆λn tan *AnδAn[* sin 1"

( "[ΔA]-1 + "[ΔZ]‰ . . . + ΔAnδ⅛

<‰h=J q ■ '2 n

Ι + {1[ΔZ, cot *A]δA* l + 2[ΔZ cot *A]δA2+. . .*

*∖ + ΔLn* cot *AnδAn}* sin 1"

Thus we have the following expression for any geodetic error—

Zt17y+ · · · + Mn^τ^n + *φ1δA1 +. . . + φnδAn = E,* ...(16),

where *μ* and *φ* represent the respective summations which are the coefficients of *δc* and *δA* in each instance but the first, in which 1 is added to the summation in forming the coefficient of *δA.*

The angular errors *x, y,* and *z* must now be introduced, in place of *δc* and *δA,* into the general expression, which will then take different forms, according as the route adopted for the line of traverse was the zigzag or the direct. In the former, the number of stations on the traverse is ordinarily the same as the number of triangles, and, whether or no, a common numerical notation may be adopted for both the traverse stations and the collateral triangles ; thus the angular errors of every triangle enter the general expression in the form *±φx +* cot *Y.μ!y -* cot *Z.Vz,*

in which *f=μ* sin 1", and the upper sign of *φ* is taken if the tri­angle lies to the left, the lower if to the right, of the line of traverse. When the direct traverse is adopted, there are only half as many traverse stations as triangles, and therefore only half the number of

*μs* and *φ's* to determine ; but it becomes necessary to adopt different numberings for the stations and the triangles, and the form of the coefficients of the angular errors alternates in successive triangles. Thus, if the *p*th triangle has no side on the line of the traverse but only an angle at the 7th station, the form is

*+ φi.xp* + cot *Yp-μ'ι-yp* - cot *Zp.μj.zp.*

If the *qth* triangle has a side between the *l*th and the (*l*+1)th sta­tions of the traverse, the form is

cot *Xq[μ'l -* √l+1)a½ + (≠i + √,+1 cot *Yq}yq - {φl+χ - μ'l* cot *Zq'}zq.*

As each circuit has a right-hand and a left-hand branch, the

errors of the angles are finally arranged so as to present equations of the general form

*[ax + by + ez]r - [ax + by + cz]i=E.*

The eleven circuit and base-line equations of condition having

been duly constructed, the next step is to find values of the angular errors whieh will satisfy these equations, and be the most probable of any system of values that will do so, and at the same time will not disturb the existing harmony of the angles in each of the seventy-two triangles. Harmony is maintained by introducing the equation of condition *x+y+z=0* for every triangle. The most probable results are obtained by the method of minimum squares, which may be applied in two ways.

(i. ) A factor λ may be obtained for each of the eighty-three equa- Γ~*?/■\* z~~~*j

tions under the condition that ——+ — is made a minimum, *u*, L *u v w* J

*V,* and *w* being the *reciprocals* of the weights of the observed angles. This necessitates the simultaneous solution of eighty-three equations to obtain as many values of λ. The resulting values of the errors of the angles in any, the *p*th, triangle, are

a⅛ = Wp[fl⅛λ] ; *yp* = ^>[^Λ] » ¾>~ Wp[Cpλ] (17).

(ii. ) One of the unknown quantities in every triangle, as *x,* may

be eliminated from each of the eleven circuit and base-line equa­tions by substituting its equivalent - *(y* + 2) for it, a similar substi­tution being made in the minimum. Then the equations take the form *[(b - a)y + (c - ay)z]= E,* while the minimum becomes

Γ(y + 2)2 y2 2°—j

Thus we have now to find only eleven values of λ by a simultaneous solution of as many equations, instead of eighty-three values from eighty-three equations ; but we arrive at more complex expressions for the angular errors as follows :—

*l/p ~ „* 4- *J4. μ ' (up ^\*^ 7⅜)[(⅛ - apl* M ~ *wp⅛cp ~* αj>)∖l} **I**

*un* ^t^ *an* -Γ *uln* I

y...(i8)∙

*~~⅝= u~~~~p+υ~~~~p~~~~p+Wp~~* \*l⅛+⅞>)[(⅞> - ⅜)M-⅜[(⅛> - ¾)λ]} J

The second method has invariably been adopted, originally be­cause it was supposed that, the number of the factors λ being re­duced from the total number of equations to that of the circuit and base-line equations, a great saving of labour would be effected. But subsequently it was ascertained that in this respect there is little to choose between the two methods ; for, when *x* is not eliminated, and as many factors are introduced as there are equations, the factors for the triangular equations may be readily eliminated at the outset. Then the really severe calculations will be restricted to the solution of the equations containing the factors for the circuit and base-line equations, as in the second method.

In the preceding illustration it is assumed that the base-lines are errorless as compared with the triangulation. Strictly speaking, however, as base-lines are fallible quantities, presumably of differ­ent weight, their errors should be introduced as unknown quantities of which the most probable values are to be determined in a simul­taneous investigation of the errors of all the facts of observation, whether linear or angular. When they are connected together by so few triangles that their ratios may be deduced as accurately, or nearly so, from the triangulation as from the measured lengths, this ought to be done ; but, when the connecting triangles are so numerous that the direct ratios are of much greater weight than the trigonometrical, the errors of the base-lines may be neglected. In the reduction of the Indian triangulation it was decided, after examining the relative magnitudes of the probable errors of the linear and the angular measures and ratios, to assume the base-lines to be errorless (see § 19, p. 704 below).

The chains of triangles being largely composed of polygons or other networks, and not merely of single triangles, as has been assumed for simplicity in the illustration, the geometrical harmony to be maintained involved the introduction of a large number of “side,” “central,” and “toto-partial” equations of condition, as well as the triangular. Thus the problem for attack was the simul­taneous solution of a number of equations of condition = that of all the geometrical conditions of every figure + four times the number of circuits formed by the chains of triangles + the number of base­lines-1, the number of unknown quantities contained in the equations being that of the whole of the observed angles ; the method of procedure, if rigorous, would be precisely similar to that