flanks, as follows:—for ratio *BC/AB,* *κ=* 1·41 and 1 ; for *CD/AB,*

1·33 and 1·23; for *AD/AB*, 2·94 and 1·99; for *AD/BC,* 2·16 and

1·46. The values for the last two ratios show that, when the length of a base-line is determined partly by measure­ment and partly by triangulation, the *p.e.* is smallest if the central section rather than an end section is measured.

If, with linear and angular *p.e*'s as in the Indian operations, a single section is measured once only, and the lengths of the other sections are derived from it by trian­gulation, the *p.e.* of the entire length will be greater than that of the whole line once measured ; it will be less if the section is measured oftener than once and the mean taken.

20. *Azimuth Observations in connexion with Principal Triangulation.—*These were invariably determined by measuring the horizontal angle between a referring mark and a circumpolar star, shortly before and after elonga­tion, and usually at both elongations in order to eliminate the error of the star’s place. Systematic changes of “face” and of the zero settings of the azimuthal circle were made as in the measurement of the principal angles (§ 9) ; but the repetitions on each zero were more numerous; the azimuthal levels were read and corrections applied to the star observations for dislevelment. As already mentioned (§ 17), the triangulation was not adjusted, in the course of the final simultaneous reduction, to the astronomically determined azimuths, because they are liable to be vitiated by local attractions ; but the azimuths observed at about fifty stations around the primary azimuthal station, which was adopted as the origin of the geodetic calculations, were referred to that station, through the triangulation, for comparison with the primary azimuth. A table was pre­pared of the differences (observed at the origin - computed from a distance) between the primary and the geodetic azi­muths ; the differences were assumed to be mainly due to the local deflexions of the plumb-line and only partially to error in the triangulation, and each was multiplied by the factor

*p=*tangent of latitude of origin /tangent of latitude of comparing station,

in order that the effect of the local attraction on the azi­muth observed at the distant station—which varies with the latitude and is = the deflexion in the prime vertical × the tangent of the latitude—might be converted to what it would have been had the station been situated in the same latitude as the origin. Each deduction was given a weight, *w,* inversely proportional to the number of tri­angles connecting the station with the origin, and the most probable value of the error of the observed azimuth at the origin was taken as

x=[(observed-computed) *p w]/[w]..............*.(24)*;*

the value of *x* thus obtained was - 1"∙1.

The formulæ employed in the reduction of the azimuth observations were as follows. In the spherical triangle *PZS,* in which *P* is the pole, *Z* the zenith, and *S* the star, the co-latitude *PZ* and the polar distance *PS* are known, and, as the angle at is a right angle at the elongation, the hour angle and the azimuth at that time are found from the equations

cos*P*=tan*PS*cot*PZ*, cos*Z*=cos*PS*sin*P*

The interval, δ*P*, between the time of any observation and that of the elongation being known, the corresponding azi­muthal angle, *8Z,* between the two positions of the star at the times of observation and elongation is given rigor­ously by the following expression—tan *8Z*

\_ 2sin2⅜δP

cotftS, sin∕,Z sin∕-, {1 + tan2PZ cosδP + *scc,2ΡS* cotP sinδ∕,} ' ’

which is expressed as follows for logarithmic computation— A m tan *Z* cos2 *PS*

*l-η+l ’*

where *m = 2* sin2 y cosec 1", n = 2 sinW sin2^, and

∕ = cotPsin *8P · lj mf* and *n* are tabulated.

21. *Calculatioη of Height and Refraction.—*Let *A* and *P* (fig. 4) be any two points the normals at which meet at *C,* cutting the sea-level at *p* and *q* ; take *Dq = Ap,* then *BD* is the difference of height ; draw the tangents *Aa* and *Bb* at *A* and *B,* then *aAB* is the depression of *B* at *A* and *bBA* that of *A* at *B;* join *AD,* then *BD* is deter­mined from the triangle *ABD.*

The triangulation gives the dis­tance between *A* and *B* at the sea- level, whence *pq = c* ; thus, putting *Ap,* the height of *A* above the sea- level, = *H,* and *pC* = *r*,

^=<1 + 7-2⅛) <26>∙

Putting *Dα* and *Db* for the actual depressions at *A* and *B, S* for the angle at *A,* usually called the “subtended angle,” and *h* for *BD—*

*S=⅛Dt-Da)* (27),

and λ=^bc⅛⅛ <28>∙

The angle at *C* being *= Db,+ Da, S* may be expressed in terms of a single vertical angle and *C* when observations have been taken at only one of the two points. *C,* the “ contained arc,” = *c* — cosec 1'' in seconds. Putting

*D'a* and *D'b* for the observed vertical angles, and *φa*, *φb* for the amounts by which they are affected by refraction, *Da = D'a+ φa* and *Db = D'b + φb∙, φa* and *φb* may differ in amount (see § 10), but as they cannot be separately ascer­tained they are always assumed to be equal ; the hypo­thesis is sufficiently exact for practical purposes when both verticals have been measured under similar atmospheric conditions. The refractions being taken equal, the ob­served verticals are substituted for the true in (27) to find *S*, and the difference of height is calculated by (28) ; the third term within the brackets of (26) is usually omitted. The mean value of the refraction is deduced from the formula

*φ= 1/2{C-(D'a+D'b)}* (29)∙

An approximate value is thus obtained from the observa­tions between the pairs of reciprocating stations in each district, and the corresponding mean “ coefficient of refrac­tion,” *φ ÷ C,* is computed for the district, and is employed when heights have to be determined from observations at a single station only. When either of the vertical angles is an elevation - *E* must be substituted for *D* in the above expressions.@@1

II. Traversing, as a basis for Survey.—Rectangular Spherical Coordinates.

Traversing is a combination of linear and angular measures in equal proportions : the surveyor proceeds from point to point, measuring the lines between them and at

@@@1 In topographical and levelling operations it is sometimes convenient to apply small corrections to observations of the height for curvature and refraction simultaneously. Putting *d* for the distance, *r* for the earth’s radius, and *κ* for the coefficient of refraction, and expressing the distance aud radius in miles and the correction to height in feet, then correction for curvature = 2/3*d*2 ; correction for refraction = *-4/3κd2;* correction for both = 2-4κ/3.*d2.*