TABLES, Mathematical. In any table the results tabulated are termed the “ tabular results ” or “ respond­ents,” and the corresponding numbers by which the table is entered are termed the “arguments.” A table is said to be of single or double entry according as there are one or two arguments. For example, a table of logarithms is a table of single entry, the numbers being the arguments and the logarithms the tabular results ; an ordinary multiplication table is a table of double entry, giving *xy* as tabular result for *x* and *y* as arguments. The intrinsic value of a table may be estimated by the actual amount of time saved by consulting it ; for example, a table of square roots to ten decimals is more valuable than a table of squares, as the extraction of the root would occupy more time than the multiplication of the number by itself. The value of a table does not depend upon the difficulty of calculating it ; for, once made, it is made for ever, and as far as the user is concerned the amount of labour devoted to its original construction is immaterial. In some tables the labour re­quired in the construction is the same as if all the tabular results had been calculated separately; but in the majority of instances a table can be formed by expeditious methods which are inapplicable to the calculation of an individual result. This is the case with tables of a continuous quan­tity, which may frequently be constructed by differences. The most striking instance perhaps is afforded by a factor table or a table of primes ; for, if it is required to deter­mine whether a given number is prime or not, the only available method (in the absence of tables) is to divide it by every prime less than its square root or until one is found that divides it without remainder. But to form a table of prime numbers the process is theoretically simple and rapid, for we have only to range all the numbers in a line and strike out every second beginning from two, every third beginning from three, and so on, those that remain being primes. Even when the tabular results are con­structed separately, the method of differences or other methods connecting together different tabular results may afford valuable verifications. By having recourse to tables not only does the computer save time and labour but he also obtains the certainty of accuracy ; in fact, even when the tabular results are so easy to calculate that no time or mental effort would be saved by the use of a table, the certainty of accuracy might make it advantageous to employ it.

The invention of logarithms in 1614, followed immedi­ately by the calculation of logarithmic tables, revolutionized all the methods of calculation ; and the original work per­formed by Briggs and Vlacq in calculating logarithms 260 years ago has in effect formed a portion of every arith­metical operation that has since been carried out by means of logarithms. And not only has an incredible amount of labour been saved @@1 but a vast number of calculations and researches have been rendered practicable which otherwise would have been quite beyond human reach. The mathematical process that underlies the tabular method of obtaining a result may be indirect and complicated ; for example, the logarithmic method would be quite unsuitable for the multiplication of two numbers if the logarithms had to be calculated specially for the purpose and were not already tabulated for use. The arrangement of a table on the page and all typographical details—such as the shape of the figures, their spacing, the thickness and placing of the rules, the colour and quality of the paper, Ac.—are of the highest importance, as the computer has

to spend hours with his eyes fixed upon the book ; and the efforts of eye and brain required in finding the right numbers amidst a mass of figures on a page and in taking them out accurately, when the computer is tired as well as when he is fresh, are far more trying than the mechanical action of simple reading. Moreover, the trouble required by the computer to *learn* the use of a table need scarcely be considered ; the important matter is the time and labour saved by it after he has learned its use. Tables are, as a rule, intended for professional and not amateur use ; and it is of little moment whether the user who is unfamiliar with a table has to spend ten seconds or a minute in obtaining an isolated result, provided it can be used rapidly and without risk of error by a skilled computer.

In the following descriptions of tables an attempt is made to give an account of all those that a computer of the present day is likely to use in carrying out arithmet­ical calculations. Tables of merely bibliographical or historical interest are not regarded as coming within the scope of this article, although for special reasons such tables are briefly noticed in some cases. Tables relating to ordinary arithmetical operations are first described, and afterwards an account is given of the most useful and least technical of the more strictly mathematical tables, such as factorials, gamma functions, integrals, Bessel’s functions, Ac. It is difficult to classify the tables de­scribed in a perfectly satisfactory manner without prolixity, as many collections contain valuable sets belonging to a variety of classes. Nearly all modern tables are stereo­typed, and in giving their titles the accompanying date is either that of the original stereotyping or of the *tirage* in question. In tables that have passed through many editions the date given is that of the edition described. A much fuller account of general tables published previously to 1872, by the present writer, is contained in the British Association *Report* for 1873, pp. 1-175; and to this the reader is referred.

*Tables of Divisors (Factor Tables) and Tables of Primes.—*The existing factor tables extend to 9,000,000. In 1811 Chernac pub­lished at Deventer his *Cribrum Arithmeticum,* which gives all the prime divisors of every number not divisible by 2, 3, or 5 up to 1,020,000. In 1814-17 Burckhardt published at Paris his *Tables des Diviseurs,* giving the least divisor of every number not divisible by 2, 3, or 5 up to 3,036,000. The second million was issued in 1814, the third in 1816, and the first in 1817. The corresponding tables for the seventh (in 1862), eighth (1863), and ninth (1865) mil­lions were calculated by Dase and issued at Hamburg. Dase died suddenly during the progress of the work, and it was completed by Rosenberg. Dase's calculation was performed at the instigation of Gauss, and he began at 6,000,000 because the Berlin Academy was in possession of a manuscript presented by Crelle extending Burckhardt’s tables from 3,000,000 to 6,000,000. This manuscript, not having been published by 1877, was found on examination to be so inaccurate that the publication was not desirable, and accordingly the three intervening millions were calculated and published by James Glaisher, the *Factor Table for the Fourth Million* appearing at London in 1879, and those for the fifth and sixth millions in 1880 and 1883 respectively (all three mil­lions stereotyped). The tenth million, though calculated by Dase and Rosenberg, has not been published. It is in the possession of the Berlin Academy, having been presented in 1878. The nine quarto volumes *(Tables des Diviseurs,* Paris, 1814-17 ; *Factor Tables,* London, 1879-83 ; *Factoren-Tafcln,* Hamburg, 1862-65) thus form one uniform table, giving the least divisor of every number not divisible by 2, 3, or 5, from unity to nine millions. The arrange­ment of the results on the page, which is due to Burckhardt, is admirable for its clearness and condensation, the least factors for 9000 numbers being given on each page. The tabular portion of each million occupies 112 pages. The first three millions were issued separately, and also bound in one volume, but the other six millions are all separate. Burckhardt began with the second million instead of the first, as Chernac’s factor table for the first million was already in existence. Burckhardt’s first million does not supersede Chernac’s, as the latter gives all the prime divisors of numbers not divisible by 2, 3, or 5 up to 1,020,000. It occupies 1020 pages, and Burckhardt found it very accurate ; he detected only thirty-eight errors, of which nine were due to the author, the remaining twenty-nine having been caused by the slipping of type

@@@1 Referring to factor tables, Lambert wrote *(Supplementa Tabularum,* 1798, p. XV.): “Universalis finis talium tabularum est ut semel pro semper computetur quod sæpius de novo computandum foret, et ut pro omni casu computetur quod in futurum pro quovis casu compu­tatum desiderabitur. ” This applies to all tables.