table of logarithms of numbers is more compendious than in Vlacq, being similar to that of an ordinary seven-figure table, but it is not so convenient, as mistakes in taking out the differences are more liable to occur. The trigonometrical canon gives log sines, cosines, tangents, and cotangents, from 0o to 2o at intervals of one second, to 10 places, without differences, and for the rest of the quadrant at intervals of ten seconds. The trigonometrical canon is not wholly reprinted from the *Trigonometria Artificialis,* as the log­arithms for every second of the first two degrees, which do not occur in Vlacq, were calculated for the work by Lieutenant Dorfmund. Vega devoted great attention to the detection of errors in Vlacq’s logarithms of numbers, and has given several important errata lists. Μ. Lefort (*Annales de l' Observatoire de Paris,* vol. iv. ) has given a full errata list in Vlacq’s and Vega’s logarithms of numbers, obtained by comparison with the great French manuscript *Tables du Cad­astre* (see Logarithms, p. 776 ; comp. also *Monthly Notices of Roy. Ast. Soc.* for May 1872, June 1872, March 1873, and 1874, suppl. number). Vega seems not to have bestowed on the trigonometrical canon anything like the care that he devoted to the logarithms of numbers, as Gauss@@1 estimates the total of last-figure errors at from 31,983 to 47,746, most of them only amounting to a unit, but some to as much as 3 or 4. As these errors in the *Trigonometria Artificialis* still remain uncorrected, it cannot be said that a reliable ten-place logarithmic trigonometrical canon exists. The calculator who has occasion to perform work requiring ten-figure logarithms of numbers should use Vlacq’s *Arithmetica Logarithmica of* 1628, after carefully correcting the errors pointed out by Vega and Lefort. After Vlacq, Vega’s *Thesaurus* is the next best table ; and Pineto’s *Tables de Logarithmes Vulgaires à Dix Décimales, con­struites d'après un nouveau mode* (St Petersburg, 1871), though a tract of only 80 pages, may be usefully employed when Vlacq and Vega are unprocurable. Pineto’s work consists of three tables: the first, or auxiliary table, contains a series of factors by which the numbers whose logarithms are required are to be multiplied to bring them within the range of table 2 ; it also gives the loga­rithms of the reciprocals of these factors to 12 places. Table 1 merely gives logarithms to 1000 to 10 places. Table 2 gives logarithms from 1,000,000 to 1,011,000, with proportional parts to hundredths. The mode of using these tables is as follows. If the logarithm cannot be taken out directly from table 2, a factor *M* is found from the auxiliary table by which the number must be multiplied to bring it within the range of table 2. Then the logarithm can be taken out, and, to neutralize the effect of the multiplication, so far as the result is concerned, log must be added ; this quantity is there­fore given in an adjoining column to *M* in the auxiliary table. A similar procedure gives the number answering to any logarithm, another factor (approximately the reciprocal of *M)* being given, so that in both cases multiplication is used. The laborious part of the work is the multiplication by *M* ; but this is somewhat com­pensated for by the ease with which, by means of the proportional parts, the logarithm is taken out. The factors are 300 in number, and are chosen so as to minimize the labour, only 25 of the 300 consisting of three figures all different and not involving 0 or 1. The principle of multiplying by a factor which is subsequently cancelled by subtracting its logarithm is used also in a tract, con­taining only ten pages, published by MM. Namur and Mansion at Brussels in 1877 under the title *Tables de Logarithmes à 12 décimales jusqu'à 434 milliards.* Here a table is given of logarithms of numbers near to 434,294, and other numbers are brought within the range of the table by multiplication by one or two factors. The logarithms of the numbers near to 434,294 are selected for tabulation because their differences commence with the figures 100 ... and the presence of the zeros in the difference renders the inter­polation easy.

If seven-figure logarithms do not give sufficiently accurate results, it is usual to have recourse to ten-figure tables ; with one exception, there exist no tables giving 8 or 9 figures. The exception is John Newton’s *Trigonometria Britannica* (London, 1658), which gives logarithms of numbers to 100,000 to 8 places, and also log sines and tangents for every centesimal minute *(i.e.,* the nine-thousandth part of a right angle), and also log sines and tangents for the first three degrees of the quadrant to 5 places, the interval being the one-thousandth part of a degree. This table is also unique in that it gives the logarithms of the differences instead of the actual differences. The arrangement of the page now universal in seven­figure tables—with the fifth figures running horizontally along the top line of the page—is due to John Newton.

As a rule seven-figure logarithms of numbers are not published separately, most tables of logarithms containing both the logarithms of numbers and a trigonometrical canon. Babbage’s and Sang’s logarithms are exceptional and give logarithms of numbers only. Babbage, *Table of the Logarithms of the Natural Numbers from 1 to 108,000* (London, stereotyped in 1827 ; there are several tirages

of later dates), is the best for ordinary use. Great pains were taken to get the maximum of clearness. The change of figure in the middle of the block of numbers is marked by a change of type in the fourth figure, which (with the sole exception of the asterisk) is the best method that has been used. Copies of the book were printed on paper of different colours—yellow, brown, green, &c.—as it was considered that black on a white ground was a fatiguing combination for the eye. The tables were also issued with title-pages and introductions in other languages. The book is not very easy to procure now. In 1871 Mr Sang published *A New Table of Seven-place Logarithms of all Numbers from 20 000 to 200 000* (London). In an ordinary table extending from 10,000 to 100,000 the differences near the beginning are so numerous that the proportional parts are either very crowded or some of them omitted ; by making the table extend from 20,000 to 200,000 instead of from 10,000 to 100,000 the differences are halved in magnitude, while there are only one-fourth as many in a page. There is also greater accuracy. A further peculiarity of this table is that multiples of the differences, instead of proportional parts, are given at the side of the page. Typographically the table is exceptional, as there are no rules, the numbers being separated from the logarithms by reversed commas. This work was to a great extent the result of an original calculation ; see *Edinburgh Transactions,* vol. xxvi. (1871). Mr Sang proposed to publish a nine-figure table from 1 to 1,000,000, but the requisite support was not obtained. Various papers of Mr Sang’s relating to his logarithmic calculations will be found in the *Edinburgh Proceedings* subsequent to 1872. In this connexion reference should be made to Abraham Sharp’s table of logarithms of numbers from 1 to 100 and of primes from 100 to 1100 to 61 places, also of numbers from 999,990 to 1,000,010 to 63 places. These first appeared in *Geometry Improv'd . . . by A. S. Philomath* (London, 1717). They have been republished in Sherwin’s, Callet’s, and the earlier editions of Hutton’s tables. Parkhurst, *Astronomical Tables* (New York, 1871), gives logarithms of numbers from 1 to 109 to 102 places.@@2

In many seven-figure tables of logarithms of numbers the values of *S* and *T* are given at the top of the page, with *V,* the variation of each, for the purpose of deducing log sines and tangents. *S* and *T*

denote log (sinx/x) and log (tanx/x) respectively, the arguments being the number of seconds denoted by certain numbers (sometimes only the first, sometimes every tenth) in the number column on each page. Thus, in Callet’s tables, on the page on which the first number is 67200, *S* = log(sin6720'/6720) and *T* = log(tan6720'/6720) ,while the *V'*s are the variations of each for 10". To find, for example, log lo52'12"∙7, or log sin 6732"∙7, we have S=4∙6854980 and log 6732∙7 — 3∙8281893, whence, by addition, we obtain 8∙5136873; but *V* for 10" is - 2∙29, whence the variation for 12"∙7 is - 3, and the log sine required is 8∙5136870. Tables of *S* and *T* are fre­quently called, after their inventor, “Delambre’s tables.” Some seven-figure tables extend to 100,000, and others to 108,000, the last 8000 logarithms, to 8 places, being given to ensure greater accuracy, as near the beginning of the numbers the differences are large and the interpolations more laborious and less exact than in the rest of the table. The eight-figure logarithms, however, at the end of a seven-figure table are liable to occasion error ; for the computer who is accustomed to three leading figures, common to the block of figures, may fail to notice that in this part of the table there are four, and so a figure (the fourth) is sometimes omitted in taking out the logarithm. In the ordinary method of arranging a seven-figure table the change in the fourth figure, when it occurs in the course of the line, is a source of frequent error unless it is very clearly indicated. In the earlier tables the change was not marked at all, and the computer had to decide for himself, each time he took out a logarithm, whether the third figure had to be increased. In some tables the line is broken where the change occurs ; but the dislocation of the figures and the corresponding irregularity in the lines are very awkward. Babbage printed the fourth figure in small type after a change. The best method seems to be that of prefixing an asterisk to the fourth figure of each logarithm after the change, as is done in Schrön’s and many other modern tables. This is beautifully clear and the asterisk at once catches the eye. Shortrede and Sang replace 0 after a change by a *nokta* (resembling a diamond in a pack of cards). This is very clear in the case of the 0’s, but leaves unmarked the cases in which the fourth figure is 1 or 2. Babbage printed a subscript point under the last figure of each logarithm that had been increased. Schrön used a bar subscript, which,

@@@1 See his “Einige Bemerkungen zu Vega’s *Thesaurus Logarithmorum,"* in *Astronomische Nachrichten* for 1851 (reprinted in his *Werke,* vol. iii. pp. 257-264); also *Monthly Notices Roy. Ast. Soc.* for May 1873.

@@@2 Legendre *(Traité des Fonctions Elliptiques,* vol. ii., 1826) gives a table of natural sines to 15 places, and of log sines to 14 places, for every 15" of the quadrant, and also a table of logarithms of uneven numbers from 1163 to 1501, and of primes from 1501 to 10,000 to 19 places. The latter, which was extracted from the *Tables du Cadastre,* is a continuation of a table in Gardiner’s *Tables of Logarithms* (London, 1742 ; reprinted at Avignon, 1770), which gives logarithms of all numbers to 1000, and of uneven numbers from 1000 to 1143. Legendre’s tables also appeared in his *Exercices de Calcul Intégral,* vol. iii. (1816).