The state of unit mass of such a substance is known by experiment to be fully determined when its volume and pressure are given, even if (as in the case of ice in presence of water, or of water in presence of steam) part of it is in one molecular state and part in another. But, the state being determinate, so must be the temperature, and also the amount of energy which the substance contains. This consideration is insisted on by Carnot as the foundation of his investigations. In other words, before we are entitled to reason upon the relation between the heat supplied to and the work done by the working substance, Carnot says we must bring that substance, by means of a *cycle* of operations, back to precisely its primitive state as regards volume, temperature, and molecular condition.

2. *Watt's Diagram.—*Watt’s indicator-diagram (see Steam-Engine) enables us to represent our operations graphically. For if OM (fig. 1) represent the volume, at any instant, of the unit mass of working substance, MP its pres­sure, the point P is determinate and corresponds to a definite temperature, definite energy, &c. If the points of any curve, as PP', in the diagram repre­sent the successive states through which the working sub­stance is made to pass, the work done is (*loc. cit.)* repre­sented by the area MPP'M'. Hence, a cycle of operations, whose essential nature is to bring the working substance back to its primitive state, is necessarily represented by a *closed* boundary, such as PP'Q'Q, in the diagram. The area enclosed is the excess of the work done by the work­ing substance over that spent on it during the cycle. [This is positive if the closed path be described clockwise, as indicated by the arrow-heads.]

3. *Carnot’s Cycle.—*For a reason which will immediately appear, Carnot limited the operations in his cycle to two kinds, employed alternately during the expansion and during the compression of the working substance. The first of these involves change of volume *at constant temperature* ; the second, change of volume *without direct loss or gain of heat.* [In his hypothetical engine the substance was supposed to be in contact with a body kept at constant temperature, or to be entirely surrounded by non-conducting materials.] The corresponding curves in the diagram are called *isothermals,* or lines of equal temperature, and *adiabatic* lines respectively. We may consider these as having been found, for any particular working substance, by the direct use of Watt’s indicator. It is easy to see that one, and only one, of each of these kinds of lines can be found for an assigned initial state of the working substance ; also that, because in expansion at constant temperature heat must be constantly supplied, the pressure will fall off less rapidly than it does in adiabatic expansion. Thus in the diagram the adiabatic lines PQ, P'Q' cut the lines of equal temperature PP', QQ' downwards and to the right. Thus the boundary of the area PP'Q'Q does not cross itself. To determine the behaviour of the engine we have therefore only to find how much heat is taken in along PP' and how much is given out in Q'Q. Their difference is *equivalent* to the work expressed by the area PP'Q'Q.

4. *Carnot's Principle of Reversibility.—*It will be observed that each operation of this cycle is strictly *reversible ;* for instance, to take the working substance along the path P'P we should have to spend on it step by step as much work as it gave out in passing along PP', and we should thus restore to the source of heat exactly the amount of heat which the working substance took from it during the expansion. In the case of the adiabatics the work spent during compression is the same as that done during the corresponding expansion, and there is no question of loss or gain of heat directly.

If, however, a transfer of heat between the working substance and its surroundings have taken place on account of a finite difference of temperature, it is clear that such an operation is not reversible. Strictly speaking, isother­mal expansion or contraction is unattainable in practice, but it is (without limit) more closely approximated to as the operation is more slowly performed. The adiabatic condition, on the other hand, is more closely approximated to in practice the more swiftly the operation is performed. We have an excellent instance of this in the compression and dilatation of air caused by the propagation of a sound­wave.

And now we have Carnot’s invaluable proposition, *a reversible heat-engine is a perfect engine,—*perfect, that is, in the sense that no other heat-engine can be superior to it. Before giving the proof, let us see the immense con­sequences of this proposition. Reversibility is the sole test of perfection; so that all heat-engines, *whatever be the working substance,* provided only they be reversible, convert into work (under given circumstances) the same fraction of the heat supplied to them. The only circum­stances involved are the temperatures of the source and condenser. Thus we are furnished with a general principle on which to reason about transformation of heat, altogether independently of the properties of any particular substance.

The proof, as Carnot gave it on the hypothesis of the materiality of heat, is *ex absurdo.* It is as follows. Suppose a heat-engine A to be capable of giving more work from a given amount of heat than is a reversible engine B, the temperatures of source and condenser being the same for each. Use the two as a compound engine, A working direct and B reversed. By hypothesis B requires to be furnished with part only of the work given by A to be able to restore to the source the heat abstracted by A, and thus at every complete stroke of the compound engine the source has its heat restored to it, while a certain amount of external work has been done. This would be the Perpetual Motion (*q.v.).*

*5. The Basis of the Second Law of Thermodynamics.—* Carnot’s reasoning, just given, is based on the hypothesis that heat (or caloric) is indestructible, and that (under certain conditions) it does work in being let down from a higher to a lower temperature, just as does water when falling to a lower level. It is clear from several expressions in his work that Carnot was not at all satisfied with this view, even in 1824, and we have seen that he soon after­wards reached the true theory. But it is also clear that such an assumption somewhat simplifies the reasoning, for in his hypothetical heat-engine all the heat which leaves the boiler goes to the condenser, and *vice versa* in the reversed working. The precise point of Carnot’s investiga­tion where the supposed indestructibility of heat introduces error is when, after virtually saying compress from Q' to a state Q determined by the condition that the heat given out shall be exactly equal to that taken in during the expansion from P to P', he assumes that, on farther com­pressing adiabatically to the original volume, the point P will be reached and the cycle completed. J. Thomson, in 1849, rectified this by putting it in the true form :— compress from Q' to a state Q, such that subsequent adiabatic compression will ultimately lead to the state P.

We have now to consider that, if an engine (whether simple or compound) does work at all by means of heat, *less* heat necessarily reaches the condenser than left the boiler. Hence, if there be two engines A and B as before, and the joint system be worked in such a way that B constantly restores to the source the heat taken from it by